NUC TP 524

BIDYNAMIC REVERBERATION FROM THE OCEAN BOTTOM

by

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Undersea Surveillance Department

October 1976

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NAVAL UNDERSEA CENTER, SAN DIEGO, CA. 92132

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ADMINISTRATIVE STATEMENT

The project to study bidynamic reverberation from the ocean bottom was sponsored by the Naval Intelligence Support Center during the period of July 1975 to June 1976, under project number ROS59.

Released by
D. A. HANNA, Head
Signal Processing and Display Division

Under Authority of
H. A. SCHENCK, Head
Undersea Surveillance Department

REPORT DOCUMENTATION PAGE		READ INSTRUCTIONS BEFORE COMPLETING FORM
1. REPORT NUMBER NUC TP 524	2. GOVT ACCESSION NO.	3. RECIPIENT'S CATALOG NUMBER
4. TITLE (and Subtitle) BIDYNAMIC REVERBERATION FROM THE OCEAN BOTTOM		5. TYPE OF REPORT & PERIOD COVERED Research and Development July 1975 - June 1976
		6. PERFORMING ORG. REPORT NUMBER
7. AUTHOR(s) O. D. Grace		8. CONTRACT OR GRANT NUMBER(s)
9. PERFORMING ORGANIZATION NAME AND ADDRESS Naval Undersea Center San Diego, California 92132		10. PROGRAM ELEMENT, PROJECT, TASK AREA & WORK UNIT NUMBERS NISC Project Number ROS59
11. CONTROLLING OFFICE NAME AND ADDRESS		12. REPORT DATE October 1976
		13. NUMBER OF PAGES 54
14. MONITORING AGENCY NAME & ADDRESS(if different from Controlling Office)		15. SECURITY CLASS. (of this report) UNCLASSIFIED
		15a. DECLASSIFICATION/DOWNGRADING SCHEDULE

16. DISTRIBUTION STATEMENT (of this Report)

Approved for public release; distribution unlimited

17. DISTRIBUTION STATEMENT (of the abstract entered in Block 20, if different from Report)

18. SUPPLEMENTARY NOTES

19. KEY WORDS (Continue on reverse side if necessary and identify by block number)

bidynamic reverberation
ocean bottom
acoustic signals

20. ABSTRACT (Continue on reverse side if necessary and identify by block number)

An expression is developed based upon the ray-wave synthesis generalization of the Kirchoff-Helmholtz acoustic field equation. This expression represents the signal received for the case of an arbitrary narrow band transmission, moving transmitter and receiver, arbitrary transmit and receive beam patterns, a smoothly varying sound speed profile and includes scattering from a rough bottom. The mean and correlation functions of this field are computed

20. (cont.)

and used to calculate the generalized instantaneous power spectrum (GIPS) and the spectral variance of the received signal.

SUMMARY

PROBLEM

Determine time evolving spectrum statistics for acoustic signals that have reverberated from the ocean floor.

RESULTS

A theoretical investigation based upon the ray-wave synthesis of the time evolving spectrum of ocean floor reverberation has been completed. This study is applicable to the case of moving platforms, arbitrary beam patterns and a varying sound speed structure.

RECOMMENDATION

Apply the results of this study to practical ocean acoustics information transfer problems.

TABLE OF CONTENTS

1. INTRODUCTION

Acoustic echo ranging signals and acoustic communication signals that are received in the ocean frequently are distorted by reverberation generated by the interaction of the transmitted signal with the ocean bottom. Additionally, if the transmitter and receiver platforms are moving, then the Doppler shifts in frequency, induced by the platform movements, further complicate the reverberation effects on the received signal. This gives added cause to consider reverberation to be an undesirable noise component. In the usual case of an assumed known environment the reverberation statistics can be determined to enable the transmitter and receiver to be designed to combat this noise.

The problem considered here is an arbitrary but narrow band signal that is transmitted by one moving platform and received by another moving platform. The platforms are assumed to have arbitrary transmit and receive beam patterns, respectively. The sound speed structure is assumed to be spatially varying in a simple manner and the direct path signal generally follows a bending ray path to the receiver. In addition to the direct path signal, the receiver intercepts the reverberation generated by the interaction between the transmitted signal and the ocean bottom. It is assumed that the random bottom has a gaussian distribution. The time evolving spectrum of the field is determined, which reveals interesting frequency-spatial relations and provides useful estimates of the acoustic field.

This development is restricted to the case of bottom scattering to avoid the mathematical complexity imposed by a moving surface. However, it is possible that the bottom reverberation results can be applied to the case of surface reverberation, with certain qualifications concerning Doppler spreading of frequencies caused by surface movement. The effects of volume scattering are ignored also as this reverberation exhibits more complicated frequency-spatial relations than boundary scattering. This is not a serious omission when bottom reverberation is present, because volume reverberation usually is much weaker than bottom reverberation.

The analysis leading to these results appears in Chapter 2 where the developed basic field expression represents the propagation and scattering mechanisms described earlier. This development is based upon a generalization of the Kirchoff-Helmholtz integral equation[1,2] in which the Green's function kernels have been replaced by ray theory kernels. The surface field in this integral is approximated by a ray-Born approximation, i.e., the surface field is equated to the direct path ray field. The surface field derivative is approximated by assuming local specular reflection with shading, which is a slight generalization of the Kirchoff approximation. This development is patterned after that of Eckart[3], who used the conventional Helmholtz integral equation and approximated the boundary conditions via the Born and Kirchoff approximations. Eckart's boundary assumptions were later extended by Horton and Muir[4] and Horton, Mitchell and Barnard[5] to include a shading function. Additionally, his method of approximating the integral was later extended by Gulin.[6]

In each of these earlier developments, as well as in more recent ones, the primary interest has been centered on the case of specular reflection and its attendant reverberation, i.e., the scattered return from the first few Freznel zones. An exposition of much of this earlier work is given in Beckmann and Spizzichino[7] and a more recent survey is given by Fortuin.[8] The specular problem is discussed here, as well as the reverberation that follows the specular component, i.e., from the higher order Freznel zones. While this component of reverberation has not been thoroughly researched one of the early investigations of this reverberation was conducted by Halley.[9,10,11]

In Chapter 3 the mean and covariance function of both components of reverberation are found. These results give the statistical properties of the time dependent field that are needed for the determination in Chapter 4 of the time evolving "generalized instantaneous power spectrum" (GIPS)[12] and also of the spectral variance. The GIPS of the reverberation from the higher order Freznel zones reveals interesting frequency-spatial relations and represents a generalization of Halley's earlier work.

2. SCATTERING BY THE OCEAN BOTTOM

THE TRANSMITTED SIGNAL

A transmitter and receiver are assumed to be traveling, respectively, at arbitrary but constant velocities $\vec{v}_T$ and $\vec{v}_R$, as indicated in figure 1. The transmitter emits a signal

$$s(t) = m(t) \exp(-i\,\omega_0 t) \tag{1a}$$

$$m(t) = \frac{1}{2\pi}\int_{-\infty}^{\infty} M(\omega) \exp(-i\,\omega t)\, d\omega \tag{1b}$$

that propagates to the receiver along the direct ray path and also along ray paths that scatter from the bottom. The emitted signal is Doppler shifted by the motion of the transmitter and amplitude weighted by the transmitter beam pattern and the direct path signal, as well as the signal that scatters from the bottom, retains this information. The received signal is Doppler shifted further by the motion of the receiver and weighted by the receiver beam. An expression is developed for the received signal in which the effect of each of these mechanisms is apparent. For mathematical convenience the case of scattering by the moving surface is ignored; i.e., only the case of scattering by a stationary bottom is considered. However, the bottom is assumed to be acoustically penetrable with scattering coefficient $R(\vec{\xi}_p)$ and it is assumed that for the appropriate scattering coefficient the development approximately applies to the case of surface scattering.

THE HARMONIC FIELD

The field in the medium due to a harmonic transmission at frequency ω is given by[1,2]

$$\phi_\omega(\vec{r}) = \frac{1}{4\pi}\Bigg\{ \int_V dv\, [g\omega(\vec{\xi}) + \phi_\omega(\vec{\xi})\ \gamma(\vec{\xi},\vec{r})] + \int_S ds\, \vec{n} \cdot [\phi_\omega(\vec{\xi})\nabla - \nabla\,\phi_\omega(\vec{\xi})] \Bigg\} \chi(\vec{\xi},\vec{r}) \tag{2a}$$

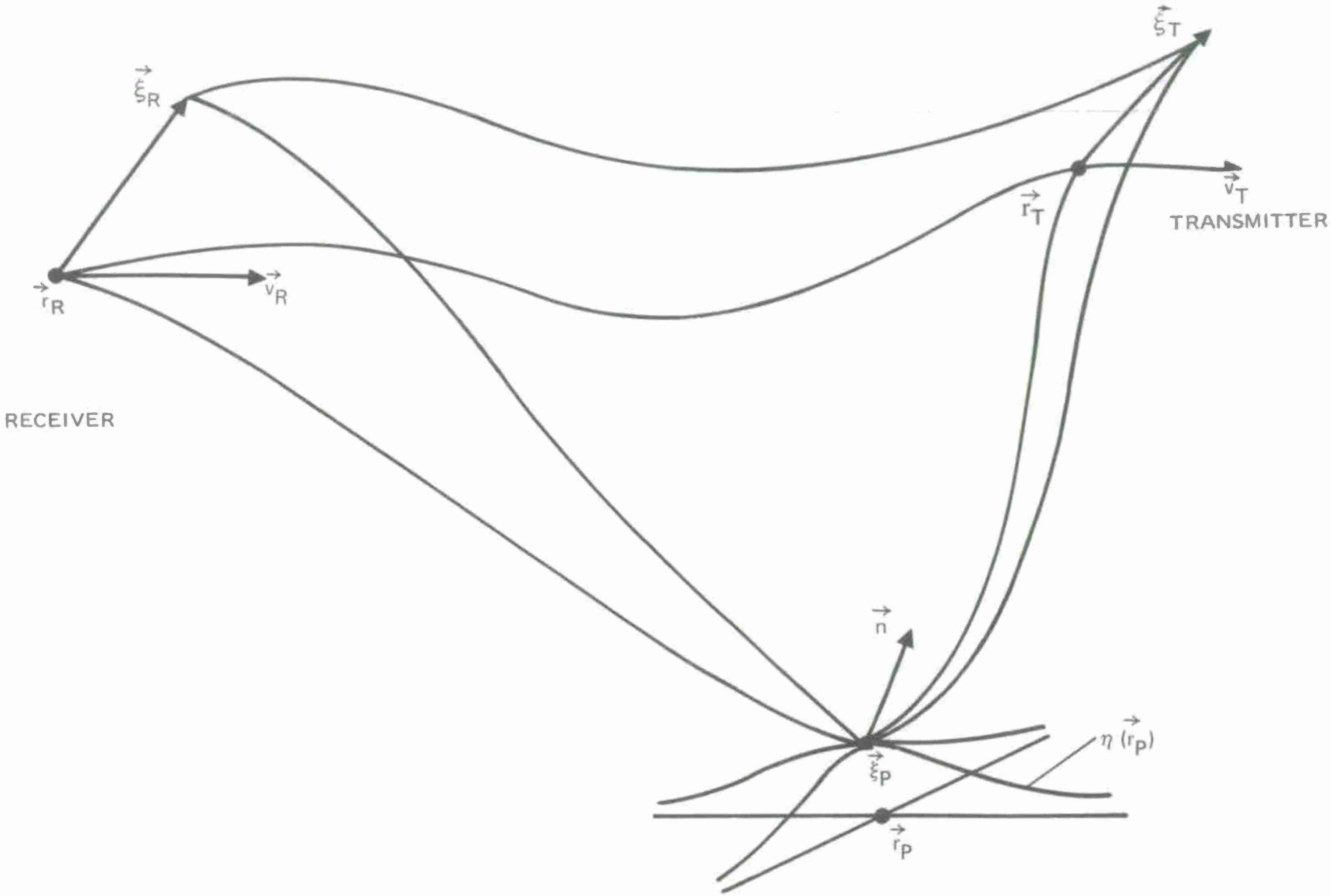

Figure 1. Ray geometry from transmitter to receiver.

$$\chi(\vec{\xi},\vec{r}) = A(\vec{\xi},\vec{r}) \exp\left(i\,k\,\psi(\vec{\xi},\vec{r})\right) \tag{2b}$$

$$\psi(\vec{\xi},\vec{r}) = \int_{\vec{r}}^{\vec{\xi}} ds\, n \tag{2c}$$

$$A(\vec{\xi},\vec{r}) = \lim_{\vec{r}\,' \to \vec{r}} \exp\left\{-\left[\frac{1}{2}\left(\int_{\vec{r}\,'}^{\vec{\xi}} ds\, \nabla^2\, \psi/n + \ln|\vec{r}\,' - \vec{r}\,|\right)\right]\right\} \tag{2d}$$

$$\gamma(\vec{\xi},\vec{r}) = \nabla^2\, A(\vec{\xi},\vec{r})/A(\vec{\xi},\vec{r}) \tag{2e}$$

where "$\cdot$" denotes a vector dot product, the time dependence in equation 2a, $\exp(-i\,\omega t)$, is omitted for convenience and the ray kernel is defined in equations 2b, c and d. The quantity $n(= c_0/c)$ is the index of refraction, where c is the sound speed and c_0 is an arbitrary reference sound speed, the quantity $k(= \omega/c_0)$ is the wave number; the integrals in equation 2c and equation 2b are over ray paths between the points $\vec{r}$ and $\vec{\xi}$.

For the case of a constant sound speed structure, the phase expression in equation 2c becomes the range, r, the amplitude expression in equation 2b becomes the reciprocal of range, l/r, and the function γ vanishes, which implies that there is no volume scattering. It is assumed that the sound speed is varying sufficiently slowly so the volume scattering term in equation 2a can be ignored. This formalism is strictly valid only for the case of a nonmoving transmitter and receiver but for velocities $\vec{v}$ that are small compared to the sound speed, the associated error terms are of the order $|\vec{v}|/c$ and can be ignored.

Initially the transmission is considered from one element of the transmitter array to one element of the receiver array. The transmitter source term is given by

$$g_\delta(\vec{\xi}) = 4\pi\, \delta(\vec{\xi} - \vec{\xi}_T)\; \Gamma_{T\omega}(\vec{\xi}_T) \tag{3}$$

where $\delta(\cdot)$ denotes the delta function and $\Gamma_{T\omega}(\vec{\xi}_T)$ denotes the frequency dependent complex transmitter transfer function. The field at an element of the receiver array due to this transmission is

$$\begin{aligned} \phi_\omega(\vec{\xi}_R, \vec{\xi}_T) = {} & \Gamma_{T\omega}(\vec{\xi}_T)\, \chi(\vec{\xi}_T, \vec{\xi}_R) \\ & + \frac{1}{4\pi} \int ds\, \vec{n} \cdot [\phi_\omega(\vec{\xi}_P, \vec{\xi}_T)\, \nabla - \nabla\, \phi_\omega(\vec{\xi}_P, \vec{\xi}_T)] \\ & \chi(\vec{\xi}_P, \vec{\xi}_R) \end{aligned} \tag{4}$$

where the transmitter, receiver and surface scattering points are denoted explicitly by $\vec{\xi}_T$, $\vec{\xi}_R$ and $\vec{\xi}_P$. This is an integral equation which can be evaluated to first order by a ray-Born approximation and a modified Kirchoff approximation.

Boundary Conditions

The field on the scattering surface can be expressed as the sum of the incident and reflected fields

$$\phi_\omega(\vec{\xi}_P,\vec{\xi}_T) = \phi^i_\omega(\vec{\xi}_P,\vec{\xi}_T) + \phi^r_\omega(\vec{\xi}_P,\vec{\xi}_T) \; . \tag{5}$$

The incident field is approximated by the shadowed direct path signal while the reflected field is approximated by the associated specular reflection.

The amplitude and phase of the specular component will be the same as the incident component except as they are modified by the reflection process and the total field on the surface can be written

$$\phi_\omega(\vec{\xi}_P,\vec{\xi}_T) = \Pi(\vec{\xi}_P,\vec{\xi}_T)\, \Gamma_{T\omega}(\vec{\xi}_T)\, \chi(\vec{\xi}_T,\vec{\xi}_P)(1 + R(\vec{\xi}_P)) \tag{6}$$

where Π is the shadowing function which is unity on the ensonified areas and zero on the shadowed areas.

Similarly, the field gradient term is given by

$$\begin{aligned} \vec{n} \cdot \nabla \phi_\omega(\vec{\xi}_P,\vec{\xi}_T) &= \vec{n} \cdot \nabla[\phi^i(\vec{\xi}_P,\vec{\xi}_T) + \phi^r(\vec{\xi}_P,\vec{\xi}_T)] \\ &= i\,k\; \Pi(\vec{\xi}_P,\vec{\xi}_T)\, \Gamma_{T\omega}(\vec{\xi}_T)\, \chi(\vec{\xi}_T,\vec{\xi}_P) \\ &\quad \times \vec{n} \cdot [\nabla\psi^i(\vec{\xi}_P,\vec{\xi}_T) + R(\vec{\xi}_P)\, \nabla\psi^r(\vec{\xi}_P,\vec{\xi}_T)] \end{aligned} \tag{7}$$

where the amplitude gradient terms are much less than the phase gradient terms and are ignored. Since the phase expression in equation 2c is symmetrical, the gradient of the phase of the incident component is approximated from the direct path phase

$$\begin{aligned} \nabla\psi^i(\vec{\xi}_P,\vec{\xi}_T) &= \nabla\psi(\vec{\xi}_T,\vec{\xi}_P) \\ &= \nabla\psi(\vec{\xi}_P,\vec{\xi}_T) \\ &= n(\vec{\xi}_P)\,\hat{e}(\vec{\xi}_P,\vec{\xi}_T) \end{aligned} \tag{8}$$

where $\hat{e}(\vec{\xi}_P,\vec{\xi}_T)$ is the unit vector that is directed along the ray from the transmitter to the surface at $\vec{\xi}_P$.

The gradient of the phase of the specular component immediately above the scattering plane can be found from the condition that its vector wave number has the same projection in the scattering surface as the incident component

$$\vec{n} \times (\nabla\psi^i + \nabla\psi^r) = 0 \; . \tag{9}$$

Since at the scattering surface the vector wave numbers have the same magnitude

$$|\nabla\psi^i| = |\nabla\psi^r| \tag{10}$$

their projections perpendicular to the scattering surface are also the same

$$\vec{n} \cdot (\nabla \psi^{i} + \nabla \psi^{r}) = 0. \tag{11}$$

With this result the field gradient expression becomes

$$\vec{n} \cdot \nabla \phi_{\omega}(\vec{\xi}_P, \vec{\xi}_T) = i\,k \;\; \Pi(\vec{\xi}_P,\vec{\xi}_T) \;\; \Gamma_{T\omega}(\vec{\xi}_T)\, \chi(\vec{\xi}_T,\vec{\xi}_P)$$
$$\times\, \vec{n} \cdot \hat{e}(\vec{\xi}_P,\vec{\xi}_T)\, (1-R(\vec{\xi}_P)) \tag{12}$$

where the index of refraction is ignored since it is of the order of unity.

The gradient of the ray kernel is similarly given by

$$\vec{n} \cdot \nabla \;\; \chi(\vec{\xi}_P,\vec{\xi}_T) = i\,k\; \chi(\vec{\xi}_P,\vec{\xi}_R)\; \vec{n} \cdot \hat{e}(\vec{\xi}_P,\vec{\xi}_R) \tag{13}$$

where $\hat{e}(\vec{\xi}_P,\vec{\xi}_R)$ is the unit vector that is directed along the ray from the receiver to the surface at $\vec{\xi}_P$.

Collecting terms the expression for the field at the receiver element becomes

$$\phi_{\omega}(\vec{\xi}_R,\vec{\xi}_T) = \Gamma_{T\omega}(\vec{\xi}_T)\, \chi(\vec{\xi}_T,\vec{\xi}_R)$$
$$+ \frac{ik}{4\pi} \int ds\; \Pi(\vec{\xi}_P,\vec{\xi}_T)\; \Gamma_{T\omega}(\vec{\xi}_T)\, \chi(\vec{\xi}_T,\vec{\xi}_P)\; \chi(\vec{\xi}_P,\vec{\xi}_R)$$
$$\times\, \vec{n} \cdot [\hat{e}(\vec{\xi}_P,\vec{\xi}_R)(1+R(\vec{\xi}_P)) - \hat{e}(\vec{\xi}_P,\vec{\xi}_T)(1-R(\vec{\xi}_P))] \;\cdot \tag{14}$$

By Huygen's principle (see appendix A) the surface integral over just the direct path term vanishes, i.e., the integral vanishes for R = 0, and this expression is simplified to the form

$$\phi_{\omega}(\vec{\xi}_R,\vec{\xi}_T) = \Gamma_{T\omega}(\vec{\xi}_T)\, \chi(\vec{\xi}_T,\vec{\xi}_R)$$
$$+ \frac{ik}{4\pi} \int ds\; \Pi(\vec{\xi}_P,\vec{\xi}_T)\; \Gamma_{T\omega}(\vec{\xi}_T)\, \chi(\vec{\xi}_T,\vec{\xi}_P)\; \chi(\vec{\xi}_P,\vec{\xi}_R)\; R(\vec{\xi}_P))$$
$$\times\, \vec{n} \cdot [\hat{e}(\vec{\xi}_P,\vec{\xi}_R) + \hat{e}(\vec{\xi}_P,\vec{\xi}_T)] \tag{15}$$

which is a first order approximation of the field at the receiver elements at $\vec{\xi}_R$ due to a harmonic transmission from the transmitter element at $\vec{\xi}_T$.

Beamformed Signals

The beamformed receiver output caused by the beamformed harmonic signal transmission can be obtained by summing over the transmitter and receiver apertures. For this calculation it is assumed that the ranges between the arrays and the bottom are large compared to both the array sizes and bottom elevations and approximations are obtained for the phases and amplitudes.

The phase of the direct path signal is expressed

$$\psi(\vec{\xi}_T,\vec{\xi}_R) = \int_{\vec{\xi}_R}^{\vec{\xi}_T} d\vec{u} \cdot n\hat{e}(\vec{u},\vec{\xi}_R) \tag{16}$$

where $\vec{u}$ denotes an arbitrary path between $\vec{\xi}_R$ and $\vec{\xi}_T$. This path is chosen via the ray path to the acoustic center of the transmitter, $\vec{r}_T$.

$$\psi(\vec{\xi}_T,\vec{\xi}_R) = \int_{\vec{r}_T}^{\vec{\xi}_T} d\vec{u} \cdot n\hat{e}(\vec{u},\vec{\xi}_R) + \psi(\vec{r}_T,\vec{\xi}_R) \quad . \tag{17}$$

Since the integrand is nearly constant for ranges that are large compared to the array sizes, the phase approximated is.

$$\psi(\vec{\xi}_T,\vec{\xi}_R) = \hat{e}_{TR} \cdot (\vec{\xi}_T - \vec{r}_T) + \psi(\vec{r}_T,\vec{\xi}_R) \tag{18}$$

where $\hat{e}_{TR}$ $(= \hat{e}(\vec{r}_T,\vec{r}_R) \cong \hat{e}(\vec{r}_T,\vec{\xi}_R))$ is the unit transmitter polar vector directed along the ray from the receiver acoustic center to the transmitter acoustic center and $n \cong 1$. Identical arugments yield the expression

$$\psi(\vec{\xi}_R,\vec{\xi}_T) = \hat{e}_{RT} \cdot (\vec{\xi}_R - \vec{r}_R) + \psi(\vec{r}_R,\vec{r}_T) \tag{19}$$

where $\hat{e}_{RT}$ is the unit receiver polar vector directed along the ray from the transmitter acoustic center to the receiver acoustic center. By symmetry of the phase function equation 18 becomes

$$\psi(\vec{\xi}_T,\vec{\xi}_R) = \hat{e}_{TR} \cdot (\vec{\xi}_T - \vec{r}_T) + \hat{e}_{RT} \cdot (\vec{\xi}_R - \vec{r}_R) + \psi(\vec{r}_T,\vec{r}_R) \quad . \tag{20}$$

The amplitude of the direct path signal is approximated

$$A(\vec{\xi}_T,\vec{\xi}_R) = A(\vec{r}_T,\vec{r}_R) \tag{21}$$

since the field expression is relatively insensitive to amplitude variations.

Similarly, for the phase and amplitude expressions within the integral

$$\psi(\vec{\xi}_T,\vec{\xi}_P) = \hat{e}_{TP} \cdot (\vec{\xi}_T-\vec{r}_T) + \hat{e}_{PT} \cdot \vec{k}\ \eta(\vec{r}_P) + \psi(\vec{r}_T,\vec{r}_P) \tag{22a}$$

$$A(\vec{\xi}_T,\vec{\xi}_P) = A(\vec{r}_T,\vec{r}_P) \tag{22b}$$

$$\psi(\vec{\xi}_P,\vec{\xi}_R) = \hat{e}_{PR} \cdot \vec{k}\ \eta(\vec{r}_P) + \hat{e}_{RP} \cdot (\vec{\xi}_R-\vec{r}_R) + \psi(\vec{r}_P,\vec{r}_R) \tag{22c}$$

$$A(\vec{\xi}_P,\vec{\xi}_R) = A(\vec{r}_P,\vec{r}_R) \tag{22d}$$

where η denotes the surface elevation, $\vec{r}_P$ denotes the projection of the surface point $\vec{\xi}_P$ onto the plane of the mean surface and the vectors $\hat{e}_{uP}$ and $\hat{e}_{Pu}$ are the unit polar vectors at $\vec{r}_u$ and $\vec{r}_P$, respectively, directed along the rays from $\vec{r}_P$ and $\vec{r}_u$.

Collecting the linearized phase expression and substituting into equation 15, along with the variable change

$$ds = dxdy/\vec{n} \cdot \vec{k} \tag{23a}$$

$$\vec{n} = (z-\eta)/\,|\,\nabla(z-\eta)\,|\ , \tag{23b}$$

multiplying the complex receiver transfer function $\Gamma_{R\omega}(\vec{\xi}_R)$ and integrating over the apertures

$$\begin{aligned} \phi_\omega(\vec{r}_R,\vec{r}_T) = {} & p_\omega(-\hat{e}_{TR})\ p_\omega(-\hat{e}_{RT})\ A(\vec{r}_T,\vec{r}_R)\ \exp(i\ k\ \psi(\vec{r}_T,\vec{r}_R)) \\ & + \frac{ik}{4\pi} \int dxdy\ \mathrm{II}(\vec{r}_P,\vec{r}_T)\ A(\vec{r}_T,\vec{r}_P)\ A(\vec{r}_P,\vec{r}_R)\ R(\vec{r}_P) \\ & \quad \times\ p_\omega(-\hat{e}_{TP})\ p_\omega(-\hat{e}_{RP}) \nabla(z-\eta(\vec{r}_P))(\hat{e}_{PT}+\hat{e}_{PR}) \\ & \quad \times\ \exp(i\ k[\,\psi(\vec{r}_T,\vec{r}_P) + \psi(\vec{r}_P,\vec{r}_R) \\ & \qquad + \eta(\vec{r}_P)\ \vec{k} \cdot (\hat{e}_{PT}+\hat{e}_{PR})\,]\,) \end{aligned} \tag{24}$$

$$p_\omega(-\hat{e}_{Tu}) = \int d\vec{\xi}_T\ \Gamma_{T\omega}(\vec{\xi}_T)\ \exp(i\ k\ \hat{e}_{Tu} \cdot (\vec{\xi}_T-\vec{r}_T)) \tag{25a}$$

$$p_\omega(-\hat{e}_{Ru}) = \int d\vec{\xi}_R\ \Gamma_{R\omega}(\vec{\xi}_R)\ \exp(i\ k\ \hat{e}_{Ru} \cdot (\vec{\xi}_R-\vec{r}_R)) \tag{25b}$$

where the functions II and R were redefined to be functions of the mean plane coordinate and the function II is approximated as a function of the transmitter acoustic center. The functions $p_\omega(-\hat{e}_{Tu})$ and $p_\omega(-\hat{e}_{Ru})$ represent the complex beam patterns of the transmitter and receiver beams, respectively, and the beam "look" direction is defined to be opposed to the unit vector arguments, i.e., back along the rays. For the arguments $\hat{e}_{TR}$ and $\hat{e}_{RT}$ the complex beam weighting is given in the direction of the ray that propagates between the transmitter and receiver. The beam patterns in the surface integral with arguments $\hat{e}_{TP}$ and $\hat{e}_{RP}$ represent the cross sections of the complex beam patterns of the transmitter and receiver that lie in the scattering surface.

THE PULSED FIELD

Given the above expression for the beamformed output due to a beamformed harmonic transmission, the analogous expression for pulses is obtained by a Fourier transformation, provided that the spatial length of the pulse is long compared to the array dimensions and bottom elevations. The Fourier transform of the complex envelope of the transmitted signal is denoted in equation 1b by $M(\omega)$ and the Fourier transform of the analytic signal is $M(\omega-\omega_0)$, where ω_0 is the angular carrier frequency. The received signal is given by

$$
\begin{aligned}
\phi(\vec{r}_T,\vec{r}_R,t) = {} & p(-\hat{e}_{TR})\, p(-\hat{e}_{RT})\, A(\vec{r}_T,\vec{r}_R) \\
& \times m(t-\psi(\vec{r}_T,\vec{r}_R)/c_0) \\
& \times \exp[-i\,\omega_0(t-\psi(\vec{r}_T,\vec{r}_R)/c_0)] \\
& + \frac{ik_0}{4\pi} \int dxdy\, \Pi(\vec{r}_P,\vec{r}_T)\, A(\vec{r}_T,\vec{r}_P)\, A(\vec{r}_P,\vec{r}_R)\, R(\vec{r}_P) \\
& \times p(-\hat{e}_{TP})\, p(-\hat{e}_{RP}) \nabla(z-\eta(\vec{r}_P)) \cdot (\hat{e}_{PT}+\hat{e}_{PR}) \\
& \times m(t-(\psi(\vec{r}_T,\vec{r}_P) + \psi(\vec{r}_P,\vec{r}_R))/c_0) \\
& \times \exp[-i\,\omega_0(t-(\psi(\vec{r}_T,\vec{r}_P) + \psi(\vec{r}_P,\vec{r}_R))/c_0) \\
& \qquad + i\,k_0\,\eta(\vec{r}_P)\,\vec{k} \cdot (\hat{e}_{PT}+\hat{e}_{PR})] \ .
\end{aligned}
\tag{26}
$$

In this Fourier transformation the beam patterns are assumed to be constant over the band of the signal, the integral coefficient is approximated by $k_0(=\omega_0/c_0)$, which is valid for narrow band signals, and the small order term in the complex envelope is ignored. This expression represents the field in the medium in terms of a broad class of signals, beam patterns, sound speed structures and bottoms for the "frozen" case and in the following section the effects of platform movement are included.

PLATFORM MOVEMENT

For the case of a moving transmitter and receiver the platform position vectors, $\vec{r}_T(t)$ and $\vec{r}_R(t)$, are assumed to be functions of time while the motion of the medium is ignored for simplicity. The surface motion has the effect of Doppler broadening of spectral components while the internal motions of the mediums have the effect of Doppler broadening and biasing of spectral components. Both effects are assumed to be negligible compared to the Doppler shifts due to the transmitter and receiver motions.

For the moving transmitter and receiver, the direct path phase is given by

$$\psi(\vec{r}_T(\tau),\vec{r}_R(t)) = \int_{\vec{r}_R(\tau)}^{\vec{r}_T(\tau)} d\vec{u} \cdot n\,\hat{e}(\vec{u},\vec{r}_R(t)) \tag{27a}$$

$$\tau = t - \psi(\vec{r}_T(\tau),\vec{r}_R(t))/c_0 \tag{27b}$$

where τ is the time at which the received signal was transmitted. If the travel distance of the platforms is short compared to the range between platforms during the interval of a pulse it can be linearized

$$\begin{aligned}\psi(\vec{r}_T(\tau),\vec{r}_R(t)) &= \int_{\vec{r}_T(0)}^{\vec{r}_T(\tau)} d\vec{u} \cdot n\,\hat{e}(\vec{u},\vec{r}_R(t)) + \psi(\vec{r}_T(0),\vec{r}_R(t)) \\ &= \hat{e}(\vec{r}_T(0),\vec{r}_R(t)) \cdot \vec{v}_T\tau + \psi(\vec{r}_T(0),\vec{r}_R(t)) \;\cdot\end{aligned} \tag{28a}$$

The second term is found from the symmetrical relation

$$\begin{aligned}\psi(\vec{r}_R(t),\vec{r}_T(0)) &= \int_{\vec{r}_R(t_D)}^{\vec{r}_R(t)} d\vec{u} \cdot n\,\hat{e}(\vec{u},\vec{r}_T(0)) + \psi(\vec{r}_R(t_D),\vec{r}_T(0) \\ &= \hat{e}(\vec{r}_R(t_D),\vec{r}_T(0)) \cdot \vec{v}_R(t-t_D) + \psi(\vec{r}_R(t_D),\vec{r}_T(0))\end{aligned} \tag{28b}$$

$$t_D = \psi(\vec{r}_T(0),\vec{r}_R(t_D))/c_0 \tag{28c}$$

where t_D is the arrival time via the direct path of the transmission at $\tau=0$. Terms collected are

$$\psi(\vec{r}_T(\tau),\vec{r}_R(t)) = \hat{e}_{TR}\cdot\vec{v}_T\tau + \hat{e}_{RT}\cdot\vec{v}_R(t-t_D)$$

$$+\ \psi(\vec{r}_T(0),\vec{r}_R(t_D)) \qquad (29a)$$

$$\tau = (t-t_D)\left[\frac{1-\hat{e}_{RT}\cdot\vec{v}_R/c_0}{1+\hat{e}_{TR}\cdot\vec{v}_T/c_0}\right] \qquad (29b)$$

where the unit polar vectors $\hat{e}_{TR}$ $(=\hat{e}(\vec{r}_T(0),\vec{r}_R(t_D))\cong\hat{e}(\vec{r}_T(0),\vec{r}_R(t)))$ and $\hat{e}_{RT}(=\hat{e}(\vec{r}_R(t_D),\vec{r}_T(0))$ are generalized to denote the ray directions at the transmitter position when the transmission starts and on the receiver position when the reception begins, respectively. This occurs when the receiver is at its position of initial reception or the transmitter at its position of initial transmission.

Similarly, for the phase in the scattering integral

$$\psi(\vec{r}_T(\tau),\vec{r}_P) + \psi(\vec{r}_P,\vec{r}_R(t))$$

$$= \int_{\vec{r}_P}^{\vec{r}_T(\tau)} d\vec{u}\cdot n\,\hat{e}(\vec{u},\vec{r}_P) + \int_{\vec{r}_R(t)}^{\vec{r}_P} d\vec{u}\cdot n\,\hat{e}(\vec{u},\vec{r}_R,(t)) \qquad (30a)$$

$$\tau = t - [\psi(\vec{r}_T(\tau),\vec{r}_P) + \psi(\vec{r}_P,\vec{r}_R(t))]/c_0 \qquad (30b)$$

where τ is the time at which the signal received via $\vec{r}_P$ was transmitted. These expressions can be approximated by

$$\psi(\vec{r}_T(\tau),\vec{r}_P) + \psi(\vec{r}_P,\vec{r}_R(t))$$

$$= \hat{e}_{TP}\cdot\vec{v}_T\tau + \hat{e}_{RP}\cdot\vec{v}_R(t-t_{PR}) + \psi(\vec{r}_T(0),\vec{r}_P) + \psi(\vec{r}_P,\vec{r}_R(t_{PR})) \qquad (31a)$$

$$\tau = (t-t_{PR})\left[\frac{1-\hat{e}_{RP}\cdot\vec{v}_R/c_0}{1+\hat{e}_{TP}\cdot\vec{v}_T/c_0}\right] \qquad (31b)$$

$$t_{PR} = [\psi(\vec{r}_T(0),\vec{r}_P) + \psi(\vec{r}_P,\vec{r}_R(t_{PR}))]/c_0\ . \qquad (31c)$$

where the subscript R on the time delay, t_{PR}, emphasizes that the receiver position at the time of reception is different for different scattering points. A first order expansion about the receiver position at the onset of the bottom return, $\vec{r}_R(t_P)$, i.e., from the specular point, gives

$$\psi(\vec{r}_P,\vec{r}_R(t_{PR})) = \psi(\vec{r}_P,\vec{r}_R(t_P)) + \hat{e}_{RP} \cdot \vec{v}_R(t_{PR}-t_P) \tag{31d}$$

where the unit vector $\hat{e}_{RP}(= \hat{e}(\vec{r}(t_P),\vec{r}_P))$ is defined to be the unit vector at the receiver that is directed along the ray from $\vec{r}_P$ to the receiver location at the onset of the bottom return. Substituting in equation 31 gives

$$\begin{aligned}\psi(\vec{r}_T(\tau),\vec{r}_P) + \psi(\vec{r}_P,\vec{r}_R(t)) &= \hat{e}_{TP} \cdot \vec{v}_T\tau + \hat{e}_{RP} \cdot \vec{v}_R(t-t_P) \\ &\quad + \psi(\vec{r}_T(0),\vec{r}_P) + \psi(\vec{r}_P,\vec{r}_R(t_P))\end{aligned} \tag{32a}$$

$$\tau = (t-t_P) \left[\frac{1-\hat{e}_{RP} \cdot \vec{v}_R/c_0}{1+\hat{e}_{TP} \cdot \vec{v}_T/c_0}\right] \tag{32b}$$

$$t_P = [\psi(\vec{r}_T(0), \vec{r}_P) + \psi(\vec{r}_P,\vec{r}_R(t_P))] / c_0 \tag{32c}$$

where the two receiver displacement terms combine to give a total displacement.

The amplitude expressions are approximated

$$A(\vec{r}_T(\tau),\vec{r}_R(t)) = A(\vec{r}_T(0),\vec{r}_R(t_D)) \tag{33a}$$

$$A(\vec{r}_T(\tau),\vec{r}_P)\, A(\vec{r}_P,\vec{r}_R(t)) = A(\vec{r}_T(0),\vec{r}_P)\, A(\vec{r}_P,\vec{r}_R(t_P)) \tag{33b}$$

since the direct path term and scattering integral are relatively insensitive to small amplitude variations.

Collecting results equation 26 becomes

$$\begin{aligned}\phi(\vec{r}_R,\vec{r}_T,t) &= p(-\hat{e}_{TR})\, p(-\hat{e}_{RT})\, A(\vec{r}_T,\vec{r}_R) \\ &\quad \times m(t-t_D) \exp[-i\, \omega_0\, \alpha'_D(t-t_D)] \\ &\quad + \frac{ik_0}{4\pi} \int dxdy\, \Pi(\vec{r}_P,\vec{r}_T)\, A(\vec{r}_T,\vec{r}_P)\, A(\vec{r}_P,\vec{r}_R)\, R(\vec{r}_P) \\ &\quad \times p(-\hat{e}_{TP})\, p(-\hat{e}_{RP}) \nabla(z-\eta(\vec{r}_P)) \cdot (\hat{e}_{PT}+\hat{e}_{PR}) \\ &\quad \times m(t-t_P) \exp[-i\, \omega_0\, \alpha'_P(t-t_P) + ik_0\, \eta(\vec{r}_P)\nu'_z]\end{aligned} \tag{34a}$$

where

$$\alpha_D' = [1 - (\hat{e}_{TR} \cdot \vec{v}_T + \hat{e}_{RT} \cdot \vec{v}_R)/c_0] \tag{34b}$$

$$\alpha_P' = [1 - (\hat{e}_{TP} \cdot \vec{v}_T + \hat{e}_{RP} \cdot \vec{v}_R)/c_0] \tag{34c}$$

$$\nu_z' = \vec{k} \cdot (\hat{e}_{PT} + \hat{e}_{PR}) \tag{34d}$$

and the small order terms in the complex envelope are ignored. These equations very nearly represent the final field equations but they can be simplified further

THE RECEIVED SIGNAL

In the preceding calculations it was convenient to consider the unit ray vectors to be directed away from their corresponding source points. In the following it will be convenient to reverse their direction, i.e., to negate all unit ray vectors. Then, whereas the vector $\hat{e}_{uv}$ was defined to be the vector at $\vec{r}_u$ directed away from $\vec{r}_v$ along their adjoining ray, i.e., along the ray that appropriately leads or follows the platform movement, it is now defined to be the vector at $\vec{r}_u$ directed toward $\vec{r}_v$ along their adjoining ray, i.e., along the ray that appropriately leads or follows the platform movement.

The amplitude expressions were retained in their general functional form throughout the preceding calculations but for the following they will be approximated by the spherical spreading law

$$A(\vec{r}_T,\vec{r}_R) = 1/r \tag{35a}$$

$$A(\vec{r}_T,\vec{r}_P)\, A(\vec{r}_P,\vec{r}_R) = 1/r_T r_R \tag{35b}$$

where r is the transmitter-receiver range and r_T and r_R are the distances between the transmitter and receiver and the mean scattering plane, respectively.

With the variable change $t \rightarrow t+t_D$ equation 34 becomes

$$\begin{aligned}\phi(t) &= \frac{1}{r} p(\hat{e}_{TR})\, p(\hat{e}_{RT})\, m(t) \exp(-i\, \omega_0\, \alpha_D\, t) \\ &+ \frac{k_0}{4\pi i} \int \frac{dxdy}{r_T r_R}\, \Pi(\vec{r}_P,\vec{r}_T)\, R(\vec{r}_P)\, p(\hat{e}_{TP})\, p(\hat{e}_{RP}) \\ &\times \nabla(z-\eta(\vec{r}_P)) \cdot (\hat{e}_{PT}+\hat{e}_{PR}) \\ &\times m(t-\tau_P) \exp[-i\, \omega_0\, \alpha_P(t-\tau_P) - ik_0\, \eta(\vec{r}_P)\nu_z]\end{aligned} \tag{36a}$$

$$\alpha_D = [1 + (\hat{e}_{TR} \cdot \vec{v}_T + \hat{e}_{RT} \cdot \vec{v}_R)/c_0] \tag{36b}$$

$$\alpha_P = [1 + (\hat{e}_{TR} \cdot \vec{v}_T + \hat{e}_{RP} \cdot \vec{v}_R)/c_0] \tag{36c}$$

$$\nu_z = \vec{k} \cdot (\hat{e}_{PT} + \hat{e}_{PR}) \tag{36d}$$

$$\tau_P = t_P - t_D \tag{36e}$$

$$p(\hat{e}_{Tu}) = \int d\vec{\xi}_T \; \Gamma_{T\omega}(\vec{\xi}_T) \exp(-i\, k_0\, \hat{e}_{Tu} \cdot (\vec{\xi}_T - \vec{r}_T)) \tag{36f}$$

$$p(\hat{e}_{Ru}) = \int d\vec{\xi}_R \; \Gamma_{R\omega}(\vec{\xi}_R) \exp(-i\, k_0\, \hat{e}_{Ru} \cdot (\vec{\xi}_R - \vec{r}_R)) \tag{36g}$$

which are our principle field equations.

The functions II, R and η are assumed to be independent random variables. Their distributions are unknown but by the central limit theorem, the integral term in equation 36a has approximately a Gaussian distribution. Consequently, the total field is a finite duration narrow band signal, i.e., the direct path signal, in a nonstationary Gaussian random signal. The field is completely defined statistically by its mean value and correlation function and the Fourier transform of its correlation function gives the GIPS of the process. These functions are calculated in Chapters 3 and 4.

3. THE ACOUSTIC FIELD STATISTICS

THE MEAN FIELD

The mean value of the field is the sum of the direct path term and the mean of the bottom scattering term

$$\langle\phi\rangle = \phi_0 + \langle\phi_1\rangle \tag{37a}$$

$$\phi_0 = \frac{1}{r}\, p(\hat{e}_{TR})\, p(\hat{e}_{RT})\, m(t) \exp(-i\, \omega_0\, \alpha_D t) \qquad (37b$$

$$\begin{aligned} \langle\phi_1\rangle = \frac{k_0}{4\pi i} \int \frac{dxdy}{r_T r_P} &\langle II(\vec{r}_P, \vec{r}_T)\rangle \langle R(\vec{r}_P)\rangle \\ &\times p(\hat{e}_{TP})\, p(\hat{e}_{RP})\, \nu_z\, m(t-\tau_P) \\ &\times \exp[-i\, \omega_0\, \alpha_P (t-\tau_P)]\; Q(k_0\, \nu_z) \end{aligned} \tag{37c}$$

where the mean value of the bottom height is zero and the characteristic function is defined

$$Q(k_0\,\nu_z) \;=\; \langle \exp[-i\,k_0\,\eta\,\nu_z] \rangle \quad . \tag{38}$$

If the bottom elevations satisfy a Gaussian distribution

$$p(\eta) \;=\; (2\pi h^2)^{-1/2} \exp(-\eta^2/2h^2) \tag{39a}$$

$$h^2 = \langle \eta^2 \rangle \tag{39b}$$

then

$$Q(k_0\,\nu_z) \;=\; \exp[\tfrac{1}{2}(k_0\,h\,\nu_z)^2] \tag{40}$$

where h is the rms bottom height.

At any time t the contributions to the reflected signal come from the ensonified portions of the bottom specified by

$$0 \leqslant (t-\tau_P) \leqslant T \tag{41a}$$

$$\tau_P \;=\; t_P - t_D \tag{41b}$$

$$t_P \;=\; [\,\psi(\vec{r}_T(0),\vec{r}_P) + \psi(\vec{r}_P,\vec{r}_R(t_P))\,]\,/c_0 \tag{41c}$$

$$t_D \;=\; \psi(\vec{r}_T(0),\,\vec{r}_R(t_D)\,)/c_0 \;. \tag{41d}$$

The term t_P represents the propagation time to the intersection of an ellipsoid-like surface, with the transmitter and receiver at its focii, and the mean scattering plane, while the term t_D represents the direct path propagation time between the transmitter and receiver.

The onset of the direct path signal occurs at the time

$$t_0 \;=\; 0 \quad . \tag{42a}$$

Denoting the specular point in the mean plane $\vec{r}_{SP}$ the condition in equation 41a shows that the onset of the specular scattering occurs at the time

$$t_1 \;=\; \tau_{SP} \tag{42b}$$

and terminates at the time

$$t_2 \;=\; \tau_{SP} + T \quad . \tag{42c}$$

During the time interval $t_1 \leqslant t \leqslant t_2$ the reflected signal comes from a region within an ellipse-like boundary centered about the specular point. For the times $t > t_2$ the received signal comes from a region confined between two ellipse-like boundaries centered about the specular point. Denoting the vertical projections of the transmitter and receiver positions onto the mean plane by $\vec{r}_{TP}$ and $\vec{r}_{RP}$, the outer ellipse-like boundary passes under the transmitter and receiver at the respective times τ_{TP} and τ_{RP}, while the inner ellipse-like boundary passes under the transmitter and receiver at the respective times $(\tau_{TP} + T)$ and $(\tau_{RP} + T)$. The average times of passage are defined as

$$t_1' = (\tau_{TP} + \tau_{RP})/2 \tag{42d}$$

$$t_2' = (\tau_{TP} + \tau_{RP})/2 + T \quad . \tag{42e}$$

The ordering of these events is dependent upon the pulse length, but for signals that are short compared to the range between the platforms, say, the events are ordered

$$t_0 < t_1 < t_2 < t_1' < t_2' \quad . \tag{42f}$$

During the time interval $t_1 \leqslant t \leqslant t_2$ the reflected signal is denoted specular scattering and it comes exclusively from the region between the transmitter and receiver, i.e., the "interior" region. For wave lengths that are short compared to the ensonified region, the exponential function in equation 37c oscillates through a number of "lowest order Freznel zones" as τ_P varies over the interior region. During the time interval $t_2' \leqslant t \leqslant \infty$ the reflected signal is denoted reverberant scattering and it comes exclusively from the complimentary "exterior" region which contains the higher order Freznel zones.

Over the interior region, the radial vectors $\hat{e}_{TP}$ and $\hat{e}_{RP}$ are approximately constant, which implies that the range of Doppler shifts is small and the integrand is approximately constant. Over the exterior region, the radial vectors range over their maximum domain of 2π steradians and, for the case of moving platforms and nonconstant beam patterns, the range of Doppler shifts is maximized and the integrand exhibits its maximum variations. In the following it is convenient to consider just the cases of specular and reverberant scattering for the short pulse case and infer more complicated cases from these results.

Specular Scattering

Specular scattering is largely from a region around the specular point $\vec{r}_{SP}$ and the specular scattering integral is approximated in equation 37c

$$< \phi_S > = \frac{k_0 < \Pi_{SP} >< R_{SP} >}{4\pi i \, r_{TSP} \, r_{RSP}} p(\hat{e}_{TSP}) \, p(\hat{e}_{RSP}) \, m(t-\tau_{SP})$$

$$\times \int dx_P dy_P \, \nu_z \exp[- \frac{1}{2} (\nu_z k_0 h)^2]$$

$$\times \exp[-i \, \omega_0 \, \alpha_{SP}(t - \tau_P)] \tag{43}$$

where the functions extracted from the integral are slowly varying in the interior region and are approximated by their values at the specular point. The remaining integral is approximated by integrating a first order expansion of the exponential function over the lowest order Freznel zones.

The time delay between the transmitter and receiver via the scattering point $\vec{r}_P$ is

$$t_P = [\psi(\vec{r}_T(0), \vec{r}_P) + \psi(\vec{r}_P, \vec{r}_R(t_P))] \, / \, c_0 \tag{44}$$

where $\vec{r}_R(t_P)$ denotes the receiver position at the onset of reception of the scattered signal. This expression can be expanded about the specular point $\vec{r}_{SP}$ by

$$t_P = \left(\int_{\vec{r}_P}^{\vec{r}_T(0)} n \, \hat{e}(\vec{u}, \vec{r}_P) \, d\vec{u} + \int_{\vec{r}_R(t_P)}^{\vec{r}_P} n \, \hat{e}(\vec{u}, \vec{r}_R(t_P)) \, d\vec{u} \right) / c_0$$

$$= \left(-\int_{x_{SP}}^{x_P} (\hat{e}_{PT} + \hat{e}_{PR}) \cdot \hat{i} \, dx - \int_{Y_{SP}}^{y_P} (\hat{e}_{PT} + \hat{e}_{PR}) \cdot \hat{j} \, dy \right) / c_0$$

$$+ [\psi(\vec{r}_T(0), \vec{r}_{SP}) + \psi(\vec{r}_{SP}, \vec{r}_R(t_P))] / c_0 \tag{45}$$

where $\hat{e}_{PT}$ and $\hat{e}_{PR}$ are the unit vectors at the scattering point that are directed along the rays to the transmitter position when the transmission starts and the receiver position when the reception begins, respectively.

First order expansions of the vector products can be developed from figure 2. At the specular point

$$(\hat{e}_{SPT} + \hat{e}_{SPR}) \cdot \hat{i} = -\cos\theta + \cos\theta$$

$$= 0 \tag{46a}$$

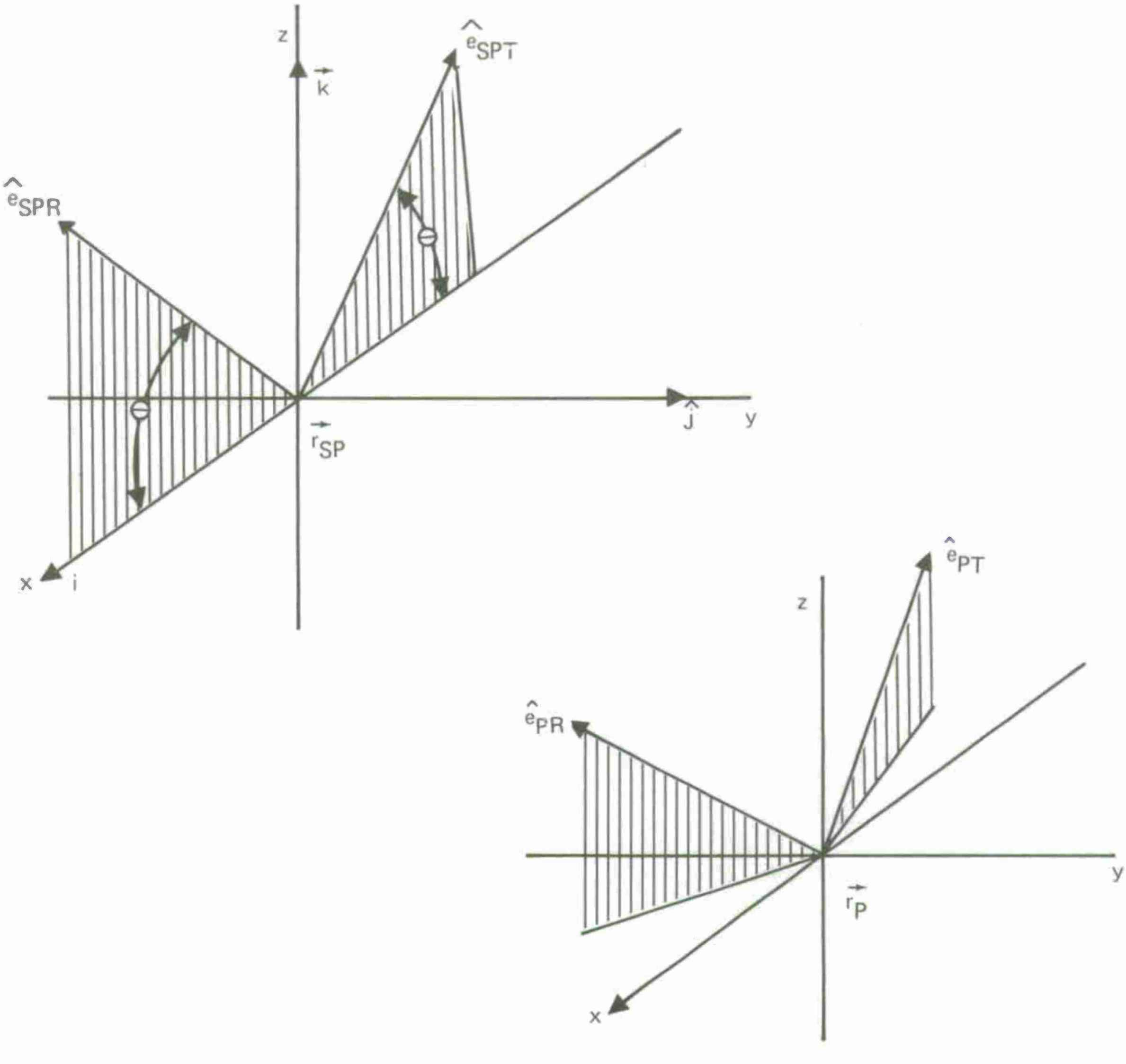

Figure 2. Ray vectors near specular point.

$$(\hat{e}_{SPT} + \hat{e}_{SPR}) \cdot \hat{j} = \cos\pi/2 + \cos\pi/2$$

$$= 0 \tag{46b}$$

$$(\hat{e}_{SPT} + \hat{e}_{SPR}) \cdot \vec{k} = \cos(\pi/2-\theta) + \cos(\pi/2-\theta)$$

$$= 2 \sin\theta \tag{46c}$$

where θ is the grazing angle. Perturbing the reflection point to $\vec{r}_P$, which is assumed to be a small displacement compared to the range, then the first order expansions

$$(\hat{e}_{PT} + \hat{e}_{PR}) \cdot \hat{i} = -\cos(\theta + \epsilon_{Tx}) + \cos(\theta + \epsilon_{Rx})$$

$$= \sin\theta\,(\epsilon_{Tx} - \epsilon_{Rx})$$

$$= -(x_P - x_{SP}) \sin^2\theta(2/R) \tag{46d}$$

$$(\hat{e}_{PT} + \hat{e}_{PR}) \cdot \hat{j} = \cos(\pi/2 + \epsilon_{Ty}) + \cos(\pi/2 + \epsilon_{Ry})$$

$$= -(\epsilon_{Ty} + \epsilon_{Ry})$$

$$= -(y_P - y_{SP})\,(2/R) \tag{46e}$$

$$(\hat{e}_{PT} + \hat{e}_{PR}) \cdot \vec{k} = \cos(\pi/2-\theta + \epsilon_{Tz}) + \cos(\pi/2-\theta + \epsilon_{Rz})$$

$$= 2 \sin\theta \tag{46f}$$

where

$$R = 2\, r_{TSP} r_{RSP} / (r_{TSP} + r_{RSP}) \ . \tag{46g}$$

The bounds on the incremental angles are denoted by double subscripts, e.g., ϵ_{Tx} is the incremental angle between the ray to the transmitter and the x-axis

Substituting these expansions about the specular points into equation 43, the integral factor becomes

$$I = \nu_{zSP} \exp[-\tfrac{1}{2}(\nu_{zSP}\, k_0\, h)^2] \exp[-i\, \omega_0\, \alpha_{SP}(t-\tau_{SP})]$$

$$\times \int dx_P dy_P \exp[i \frac{k_0 \alpha_{SP}}{R} (\sin^2\theta (x_P - x_{SP})^2 + (y_P - y_{SP})^2)] \ . \tag{47}$$

With the variable changes

$$t_1 = \sqrt{\frac{k_0\alpha_{SP}}{R}}\ \sin\theta(x_P - x_{SP})\ ;\ t_2 = \sqrt{\frac{k_0\alpha_{SP}}{R}}\ (y_P - y_{SP}) \qquad (48a,b)$$

this integral can be cast into the form of a product of Freznel integrals

$$I = \nu_{zSP} \exp[-\frac{1}{2}(\nu_{zSP}\ k_0\ h)^2]\ \exp[-i\ \omega_0\ \alpha_{SP}(t-\tau_{SP})]$$

$$\times\left(\frac{R}{k_0\alpha_{SP}\sin\theta}\right) \int dt_1\ \exp(i\ t_1^2) \int dt_2\ \exp(i\ t_2^2)$$

$$= \frac{i\pi\ \nu_{zSP}\ R}{k_0\alpha_{SP}\sin\theta}\ \exp\ [-\frac{1}{2}(\nu_{zSP}\ k_0\ h)^2]\ \exp\ [-i\ \omega_0\ \alpha_{SP}(t-t_{SP})]\ . \qquad (49)$$

Substituting back into the specular scattering integral, then

$$<\phi_{SR}> = <\Pi_{SP}><R_{SP}>\ \exp[-\frac{1}{2}(\nu_{zSP}\ k_0\ h)^2]$$

$$\times\ p(\hat{e}_{TSP})\ p(\hat{e}_{RSP})\ m(t - \tau_{SP})$$

$$\times\ \exp[-i\ \omega_0\ \alpha_{SP}(t-\tau_{SP})]\ /(r_{TSP} + r_{RSP}) \qquad (50a)$$

$$\alpha_{SP} = [1 + (\hat{e}_{TSP} \cdot \vec{n}_T + \hat{e}_{RSP} \cdot \vec{n}_R)\ /\ c_0] \qquad (50b)$$

$$\nu_{zSP} = 2\sin\theta\ \cdot \qquad (50c)$$

This is a principal result showing that the mean reflection from the near surface is a specular reflected signal attenuated by shading absorption and surface roughness. The starting and stopping times of this signal are given in equation 42b,c.

Reverberant Scattering

For the case of reverberant scattering it is convenient to represent the signal's complex envelope by the integral

$$m(t - \tau_P) = \lim_{\epsilon \to 0} \int_0^T dt'\ \ m(t')\ p_\epsilon(t-\tau_P-t') \qquad (51a)$$

$$p_\epsilon(t') = 1/\epsilon \qquad 0 \leqslant t' \leqslant \epsilon$$

$$= 0 \cdot \qquad \text{elsewhere} \tag{51b}$$

Substituting into the expression for the mean value of the reflected field in equation 37c,

$$\langle\phi_R\rangle = \lim_{\epsilon\to 0} \frac{k_0}{4\pi i} \int_0^T dt' \; m(t') \int_0^\infty \int_0^{2\pi} \frac{d\rho_T d\theta_T}{r_T r_S}$$

$$\times \langle H(\vec{r}_P,\vec{r}_T)\rangle\langle R(\vec{r}_P)\rangle \, \nu \exp[-\tfrac{1}{2}(\nu_z \, k_0 \, h)^2]$$

$$\times \; p_\epsilon(t-\tau_P-t') \; \exp[-i \, \omega_0 \, \alpha_P(t-\tau_P)] \tag{52}$$

where ρ_T, θ_T are the cylindrical coordinates in the mean scattering plane about the transmitter.

The only contribution to the inner integral at a time t satisfying equation 52 comes from an ellipse-like band defined by the condition

$$0 \leqslant (t-\tau_P-t') \leqslant \epsilon \quad \cdot \tag{53}$$

The radial coordinate integration over this band can be easily performed. For a fixed time t, angular coordinate θ_T and parameter t' the upper and lower radial coordinate limits are of the form

$$\rho_{T,u} = \rho_T(t, \theta_T, t') \tag{54a}$$

$$\rho_{T,L} = \rho_T(t, \theta_T, t' + \epsilon) \quad \cdot \tag{54b}$$

Expanding the lower limit in a Taylor series about the parameter t'

$$\rho_{T,L} = \rho_{T,u} + \left(\frac{d\rho_T}{dt'}\right) \epsilon(t,\theta_T,t') \tag{54c}$$

and the integral in equation 52 becomes

$$\langle \phi_R \rangle = \frac{k_0}{4\pi i} \int_0^T dt' \; m(t') \int_0^{2\pi} d\theta_T \; \frac{\rho_T}{r_T r_T} \langle \Pi(\vec{r}_P, \vec{r}_T) \rangle \langle R(\vec{r}_P) \rangle$$

$$\times \; p(\hat{e}_{TP}) \, p(\hat{e}_{RP}) \, \nu_z' \exp[-\tfrac{1}{2}(\nu_z k_0 h)^2]$$

$$\times \; \exp(-i\,\omega_0\,\alpha_P t') \, (-d\rho_T/dt')\,]_{(t,\theta_T,t')} \qquad (55)$$

where the integrand is evaluated at the point on the ellipse-like band (t, θ_T, t'). Since the quantities in brackets are relatively insensitive to variations in t' and become independent of t' for $t \to \infty$, the integrals can be approximately uncoupled

$$\langle \phi_R \rangle = \frac{k_0}{4\pi i} \int_0^{2\pi} d\theta_T \left[\left(\frac{\rho_T}{r_T r_P} \right) \langle \pi(\vec{r}_P, \vec{r}_T) \rangle \langle R(\vec{r}_P) \rangle \right.$$

$$\times \; p(\hat{e}_{TP}) \, p(\hat{e}_{RP}) \, \nu_z \exp[-\tfrac{1}{2}(\nu_z k_0 h)^2]$$

$$\left. \times \; (-d\,\rho_T/dt') \right]_{(t,\,\theta_T)} \int_0^T dt' \; m(t') \exp(-i\omega_0 \alpha_P t) \; . \qquad (56)$$

Since the complex envelope is bandlimited below the frequency $\omega_0 \alpha_P$

$$\phi_R = 0 \qquad (57)$$

which shows that the signal scattered from the exterior region is incoherent and has no specular component.

THE CORRELATION FUNCTION

The correlation function for the total field is given by

$$\langle \phi(t+\tau)\,\phi^*(t) \rangle = \phi_0(t+\tau)\,\phi_0{}^*(t) + \phi_0(t+\tau) \langle \phi_1{}^*(t) \rangle$$

$$+ \; \phi_0^*(t) \; \langle \phi_1(t+\tau) \rangle \; + \; \langle \phi_1(t+\tau)\,\phi_1^*(t) \rangle \qquad (58)$$

where the mean was computed in the mean field section. The correlation function of the reflected field is given from equation 36a by

$$C(t+\tau,t) = \langle \phi_1(t+\tau)\, \phi_1^*(t) \rangle$$

$$= \frac{k_0^2}{(4\pi)^2} \iiiint \frac{dx_P\, dy_P\, dx'_P\, dy'_P}{r_T\, r_R\, r'_T\, r'_R}\, C_\Pi(\vec{r}_P - \vec{r}'_P, \vec{r}_T)$$

$$\times\, C_R(\vec{r}_P - \vec{r}'_P)\, p(\hat{e}_{TP})\, p(\hat{e}_{RP})\, p^*(e_{TP'})\, p^*(e_{RP'})$$

$$\times\, C_S\, (\vec{r}_P - \vec{r}_P{}')\, m(t+\tau-\tau_P)\, m^*(t-\tau'_P)$$

$$\times\, \exp[-i\, \omega_0\, \alpha_P(t+\tau-\tau_P) + i\, \omega_0\, \alpha'_P(t+\tau-\tau'_P)] \tag{59a}$$

$$C_\Pi(\vec{r}_P - \vec{r}'_P, \vec{r}_T) = \langle \Pi(\vec{r}_P, \vec{r}_T)\, \Pi(\vec{r}'_P, r_T) \rangle \tag{59b}$$

$$C_R(\vec{r}_P - \vec{r}'_P) = \langle R(\vec{r}_P)\, R(\vec{r}'_P) \tag{59c}$$

$$C_S(\vec{r}_P - \vec{r}'_P) = \langle [\nabla(z-\eta) \cdot (\hat{e}_{PT} + \hat{e}_{PR})]\, [\nabla'(z-\eta') \cdot (\hat{e}'_{PT} + \hat{e}'_{PR})]$$

$$\times\, \exp[-i\, k_0\, \eta\nu_z + i\, k_0\, \eta'\, \nu'_z] \rangle \tag{59d}$$

where homogeneous correlations are assumed. In this expression the correlation functions C_Π, C_R and C_S are for the shading function, the reflection coefficient and the geometric scattering coefficient, respectively, and the expectation operator separates due to the independences of Π, R and η. Since the correlations vanish for $|\vec{r}_P - \vec{r}'_P| >> 0$ this integral can be reduced to a single surface integral and evaluated for the cases of scattering from the interior and exterior regions separately as in the preceding section.

The geometric scattering coefficient correlation function can be approximated, where ν_z is much more slowly varying than η,

$$C_S(\vec{r}_P - \vec{r}'_P) = \langle [\, (\hat{e}_{PT} + \hat{e}_{PR}) \cdot (\vec{k} - (-i\, k_0\, \nu_z) - \nabla)\, \exp(-i\, k_0\, \eta\nu_z)\,]$$

$$\times\, [(\hat{e}'_{PT} + \hat{e}'_{PR}) \cdot (\vec{k} - (-i\, k_0\, \nu'_z) - \nabla')\, \exp(i\, k_0\, \nu'_z\, \eta')\,] \rangle$$

$$= \Big\{\, \nu_z\nu'_z - \nu_z(i\, k_0\, \nu'_z)^{-1}\, [\, (\hat{e}'_{PT} + \hat{e}'_{PR}) \cdot \nabla'\,]$$

$$-\, \nu'_z(-i\, k_0\, \nu_z)^{-1}\, [\, (\hat{e}_{PT} + \hat{e}_{PR}) \cdot \nabla\,]$$

$$+\, (-i\, k_0\, \nu_z)^{-1}\, (i\, k_0\, \nu'_z)^{-1}\, [\, (\hat{e}_{PT} + \hat{e}_{PR}) \cdot \nabla\,][\hat{e}'_{PT} + \hat{e}'_{PR}) \cdot \nabla'\,]\, \Big\}$$

$$\times\, Q_2(k_0\, \nu_z,\, k_0\, \nu'_z,\, \vec{r}_P - \vec{r}'_P) \tag{60}$$

where the function

$$Q_2(k_0\,\nu_z,\,k_0\,\nu_z',\,\vec{r}_P-\vec{r}_P') = \langle \exp[-i\,k_0\,\eta\nu_z + i\,k_0\,\eta'\nu_z'] \rangle \tag{61}$$

is the joint characteristic function of the bottom elevations.

Assuming a jointly normal distribution for the surface the joint characteristic function becomes

$$Q_2(k_0\,\nu_z,\,k_0\,\nu_z',\,\vec{r}_P-\vec{r}_P') = \exp[-\tfrac{1}{2}(\,(k_0\,h\nu_z)^2 + (k_0\,h\nu_z')^2$$

$$-2\,(k_0\,h\nu_z)(k_0\,h\nu_z')\,C_\eta(\,|\vec{r}_P-\vec{r}_P'|)\,)\,] \tag{62a}$$

$$C_\eta(\,|\vec{r}_P-\vec{r}_P'|) = \langle \eta(\vec{r}_P)\,\eta(\vec{r}_P') \rangle\,/h^2 \tag{62b}$$

$$C_\eta(0) = 1 \tag{62c}$$

where an homogeneous isotropic bottom is assumed.

The above expression can be approximated by a first order expansion of the surface correlation function

$$C_\eta(\,|\vec{r}_P-\vec{r}_P'|) \cong [1 - |\vec{r}_P-\vec{r}_P'|/a^2],\ |\vec{r}_P-\vec{r}_P'| < a$$

$$\cong 0,\ |\vec{r}_P-\vec{r}_P'| > a\ , \tag{63}$$

i.e., the surface correlation vanishes within the distance a where a is the surface correlation distance. Since the correlation distance is short compared to the ranges from which energy is scattered, the angles to the two points $\vec{r}_P$ and $\vec{r}_P'$ can be equated.

Collecting results then

$$C_S(\vec{r}_P-\vec{r}_P') = \Big\{ \nu_z^2 + 2\nu_z(i\,k_0\,\nu_z)^{-1}\,[\,(\hat{e}_{PT}+\hat{e}_{PR})\cdot\nabla]$$

$$-\,(k_0\,\nu_z)^{-2}\,[\,(\hat{e}_{PT}+\hat{e}_{PR})\cdot\nabla]^2 \Big\}$$

$$\times \exp[-(k_0\,h\nu_z)^2\,|\vec{r}_P-\vec{r}_P'|^2/a^2] \tag{64}$$

where $\nabla' \to -\nabla$. This expression can be readily evaluated with the substitution

$$|\vec{r}_P-\vec{r}_P'|^2 = (x_P - x_P')^2 + (y_P - y_P')^2 \tag{65}$$

and the result is

$$C_S(\vec{r}_P - r'_P) = \Big\{ [\nu_z^2 + i\,4(h/a)^2\,\nu_z^2\,k_0[\,(x_P - x'_P)\,\nu_x + (y_P - y'_P)\,\nu_y]$$

$$- 4(h/a)^4\,\nu_z^2\,k_0^2\,[\,(x_P - x'_P)\nu_x + (y_P - y'_P)\nu_y]^2$$

$$+ 2(h/a)^2\,(\nu_x^2 + \nu_y^2)\Big\}$$

$$\times \exp[\,(k_0\,h\,\nu_z/a)^2\,(\,(x_P - x'_P)^2 + (y_P - y'_P)^2)\,] \tag{66a}$$

$$\nu_x = (\hat{e}_{PT} + \hat{e}_{PR})\cdot\hat{i} \tag{66b}$$

$$\nu_y = (\hat{e}_{PT} + \hat{e}_{PR})\cdot\hat{j} \tag{66c}$$

which is approximately correct for all x_P, y_P, x'_P, y'_P.

Collecting results the correlation function of the reflected field is given by

$$C(t+\tau, t) = \frac{k_0{}^2}{(4\pi)^2} \iiiint \frac{dx_P\,dy_P\,dx'_P\,dy'_P}{r_T\,r_R\,r'_T\,r'_R}\; C_\pi(\vec{r}_P - \vec{r}'_P, \vec{r}_T)\, C_R(\vec{r}_P - \vec{r}'_P)$$

$$\times\ |p(\hat{e}_{TP})\,|^2\,|p(\hat{e}_{RP})\,|^2\,m(t+\tau-\tau_P)\,m^*(t-\tau'_P)$$

$$\times \exp\,[\,i\,\omega_0\,\alpha_P\,(\tau_P - \tau'_P - \tau)\,]$$

$$\times \Big\{\nu_z^2 + i\,4(h/a)^2\nu_z^2\,k_0\,[(x_P - x'_P)\nu_x + (y_P - y'_P)\nu_y]$$

$$- 4(h/a)^4\,\nu_z^2\,k_0^2[\,(x_P - x'_P)\nu_x + (y_P - y'_P)\nu_y]^2$$

$$+ 2(h/a)^2\,(\nu_x^2 + \nu_y^2)\Big\}$$

$$\times \exp[-(k_0\,h\nu_z/a)^2\,(\,(x_P - x'_P)^2 + (y_P - y'_P)^2)\,]\ . \tag{67}$$

The integration over the primed coordinates can be performed after expanding τ'_P about τ_P.

From equation 36e then

$$\tau_P - \tau'_P = t_P - t'_P \tag{68a}$$

and analogous to the expansion in equation 45 is obtained

$$t_P - t_P' = \left(\int_{x_P}^{x'_P} (\hat{e}_{PT} + \hat{e}_{PR}) \cdot \vec{i}\, dx + \int_{y_P}^{y'_P} (\hat{e}_{PT} + \hat{e}_{PR}) \cdot \vec{i}\, dy \right) / c_0 \tag{68b}$$

where $\hat{e}_{PT}$ and $\hat{e}_{PR}$ are the unit vectors at the scattering point $\vec{r}_P$ that are directed along the rays to the transmitter and receiver, respectively.

With reference to figure 3 the expansions for the direction cosines of the rays about the point $\vec{r}_P$ are

$$(\hat{e}_{PT} + \hat{e}_{PR}) \cdot \vec{i} = \cos\theta_{Tx} + \cos\theta_{Rx} \tag{69a}$$

$$(\hat{e}_{PT} + \hat{e}_{PR}) \cdot \hat{j} = \cos\theta_{Ty} + \cos\theta_{Ry} \tag{69b}$$

$$(\hat{e}_{PT} + \hat{e}_{PR}) \cdot \vec{k} = \cos\theta_{Tz} + \cos\theta_{Rz} \tag{69c}$$

$$\begin{aligned}(\hat{e}'_{PT} + \hat{e}'_{PR}) \cdot \hat{i} &= \cos(\theta_{Tx} + \epsilon_{TX}) + \cos(\theta_{Rx} + \epsilon_{Rx}) \\ &= (\hat{e}_{PT} + \hat{e}_{PR}) \cdot \hat{i} \\ &\quad - \left(\frac{\sin^2 \theta_{Tx}}{r_T} + \frac{\sin^2 \theta_{Rx}}{r_R} \right) (x'_P - x'_P)\end{aligned} \tag{69d}$$

$$\begin{aligned}(\hat{e}'_{PT} + \hat{e}'_{PR}) \cdot j &= \cos(\theta_{Ty} + \epsilon_{Ty}) + \cos(\theta_{Ry} + \epsilon_{Ry}) \\ &= (\hat{e}_{PT} + \hat{e}_{PR}) \cdot \hat{j} \\ &\quad - \left(\frac{\sin^2 \theta_{Ty}}{r_T} + \frac{\sin^2 \theta_{Ry}}{r_R} \right) (y'_P - y'_P)\end{aligned}$$

$$\begin{aligned}(\hat{e}'_{PT} + \hat{e}'_{PR}) \cdot \vec{k} &= \cos(\theta_{Tz} + \epsilon_{Tz}) + \cos(\theta_{Rz} + \epsilon_{Rz}) \\ &= (\hat{e}_{PT} + \hat{e}_{PR}) \cdot \vec{k}\end{aligned} \tag{69f}$$

where the angular subscripts are defined as in equation 46.

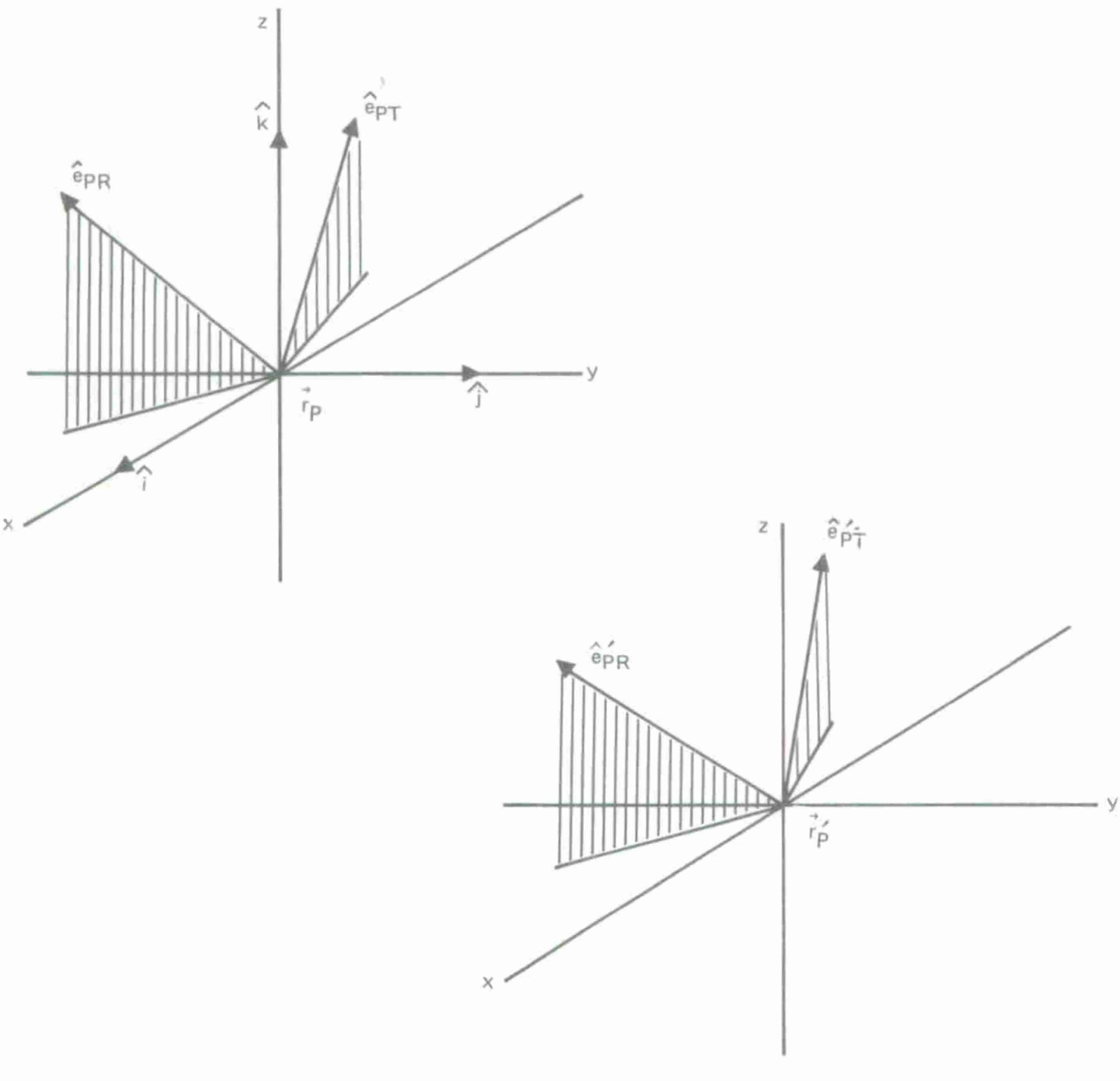

Figure 3. Ray vectors at the points $\vec{r}_{p'}$ and $\vec{r}_{p'}$.

Substituting the direction cosine expansions into the time delay expressions and integrating then

$$\tau_P - \tau'_P = (x'_P - x_P)\nu_x/c_0$$

$$- \left(\frac{\sin^2 \theta_{Tx}}{r_T} + \frac{\sin^2 \theta_{Rx}}{r_R}\right)(x'_P - x_P)^2/2\,c_0$$

$$+ (y'_P - y_P)\nu_y/c_0$$

$$- \left(\frac{\sin^2 \theta_{Ty}}{r_T} + \frac{\sin^2 \theta_{Ry}}{r_R}\right)(y'_P - y_P)^2/2\,c_0 \tag{70}$$

which is the required expansion.

The correlation function becomes

$$C(t+\tau,t) = \frac{k_0{}^2}{(4\pi)^2}\iint \frac{dx_P\, dy_P}{r_T^2\ \ r_R^2}\ |p(\hat{e}_{TP})|^2\ |p(\hat{e}_{RP})|^2$$

$$\times\ m(t+\tau-\tau_P)\ m^*(t-\tau_P)\ \exp(-i\ \omega_0\ \alpha_P\ \tau)$$

$$\times \iint d\epsilon_x d\epsilon_y\ C_\pi(\vec{\epsilon},\vec{r}_T)\ C_R(\vec{\epsilon})$$

$$\times\ \Big\{\nu_z^2 - i\ 4(h/a)^2\ \nu_z^2\ k_0[\epsilon_x\nu_x + \epsilon_y\nu_y]$$

$$- 4(h/a)^4\ \nu_z^2\ k_0^2[\epsilon_x\nu_x + \epsilon_y\nu_y]^2$$

$$+ 2(h/a)^2\ (\nu_x^2 + \nu_y^2)\Big\}$$

$$\times\ \exp\left[-(k_0\ h\nu_z/a)^2\ (\epsilon_x^2 + \epsilon_y^2)\right]$$

$$\times \exp\left[-i\ k_0\ \alpha_P\left(\epsilon_x\nu_x + \left(\frac{\sin^2 \theta_{Tx}}{r_T} + \frac{\sin^2 \theta_{Rx}}{r_R}\right)\frac{\epsilon_x^2}{2}\right.\right.$$

$$\left.\left. + \epsilon_y\nu_y + \left(\frac{\sin^2 \theta_{Ty}}{r_T} + \frac{\sin^2 \theta_{Ry}}{r_R}\right)\frac{\epsilon_y^2}{2}\right)\right] \tag{71a}$$

where

$$\vec{r}_P - \vec{r}'_P = \vec{\epsilon} = \epsilon_x \hat{i} + \epsilon_y \hat{j} \quad . \tag{71b}$$

If it is assumed that the correlation functions of the shading function and reflection coefficient are approximately constant over the correlation distance of the medium, the inner integrals become elementary integrals and the correlation integral becomes

$$C(t+\tau, t) = \frac{C_R(0)}{16\pi} \left(\frac{a}{h}\right)^2 \iint \frac{dx_P \, dy_P}{r_T^2 \; r_R^2} \, |p(\hat{e}_{TP})|^2 \quad |p(\hat{e}_{RP})|^2 \, C_\Pi(0, \vec{r}_T)$$

$$\times \; m(t+\tau - \tau_P) \, m^*(t - \tau_P) \exp(-i \, \omega_0 \, \alpha_P \, \tau)$$

$$\times \; \nu_z^{-2} \, [\nu_z^2 + \nu_z^{-2} (\nu_x^2 + \nu_y^2)^2 - 2(\nu_x^2 + \nu_y^2)]$$

$$\times \; \exp[-(a/2 \, h\nu_z)^2 \, (\nu_x^2 + \nu_y^2)] \quad . \tag{72}$$

As for the case of the mean field calculation this integral is evaluated for the cases of scattering from the interior and exterior regions separately.

Specular Scattering Correlation

For the case of near field scattering the energy is received largely from the specular point, for which $\nu_x = \nu_y = 0$, and the correlation function is approximated

$$C(t+\tau, t) = \frac{A \; C_\Pi(0, \vec{r}_{TSP}) \; C_R(0)}{16\Pi \; r_{TSP}^2 \; r_{RSP}^2} \left(\frac{a}{h}\right)^2 |p(\hat{e}_{TSP})|^2 \; |p(\hat{e}_{RSP})|^2$$

$$\times \; m(t+\tau - \tau_{SP}) \; m^*(t - \tau_{SP}) \exp(-i \; \omega_0 \; \alpha_{SP} \; \tau) \tag{73a}$$

where all functions in the integrand are approximately constant over the interior region and the quantity A is the area of the ensonified surface. A crude approximation of the area A, which is roughly the area of the interior region and that gives the correct range dependence, is given by

$$A \cong \frac{\pi \; r_{TSP}^2 \; r_{RSP}^2}{(r_{TSP} + r_{RSP})^2} \quad . \tag{73b}$$

With this expression equation 73a becomes

$$C(t+\tau,t) = \frac{C_{\Pi}(0,\vec{r}_{TSP})\; C_R(0)}{16(r_{TSP} + r_{RSP})^2} \left(\frac{a}{h}\right)^2 |p(\hat{e}_{TSP})|^2 \; |p(\hat{e}_{RSP})|^2$$

$$\times \; m(t+\tau-\tau_{SP}) \; m^*(t-\tau_{SP}) \exp(-i\,\omega_0\, \alpha_{SP}\, \tau) \tag{73c}$$

which is easily obtained and shows the functional form of the correlation.

Reverberant Scattering Correlation

For the case of scattering from the exterior region we represent the complex envelope product by the integral

$$m(t+\tau-\tau_P)\; m^*(t-\tau_P)$$

$$= \lim_{\epsilon\to 0} \int_0^T dt' \quad m(t'+\tau)\; m^*(t')\; p_\epsilon(t-\tau_P-t') \tag{74}$$

where the function p_ϵ is defined in equation 51b. Substituting into the correlation function in equation 72,

$$C(t+\tau,t) = \lim_{\epsilon\to 0} \frac{C_R(0)}{16\pi} \left(\frac{a}{h}\right)^2 \exp(a/2h)^2 \; x \int_0^T dt' \quad m(t'+\tau)\; m^*(t')$$

$$\times \iint \frac{dx_P\, dy_P}{r_T^2 \; r_R^2} \; |p(\hat{e}_{TP})|^2 \; |p(\hat{e}_{RP})|^2 C_{\Pi}(0,\vec{r}_T)$$

$$\times \left[1 + (|\hat{e}_{PT}+\hat{e}_{PR}|^2-\nu_z^2)\,\nu_z^4 - 2\,(|\hat{e}_{PT}+\hat{e}_{PR}|^2-\nu_z^2)/\nu_z^2\right]$$

$$\times \quad \exp(-i\,\omega_0\, \alpha_P\, \tau)\; p_\epsilon(t-\tau_P-t') \; x \; \exp[-(a/2\,h\nu_z)^2$$

$$|\hat{e}_{PT}+\hat{e}_{PR}|^2]$$

(75a)

where

$$\nu_x^2 + \nu_y^2 + \nu_z^2 = |\hat{e}_{PT}+\hat{e}_{PR}|^2 \quad . \tag{75b}$$

By the same argument applied in the mean field calculation the radial coordinate can be integrated to obtain

$$\begin{aligned} C(t+\tau,t) &= \frac{C_R(0)\, C_{II}(t)}{16\pi} \left(\frac{a}{h}\right)^2 \exp\,(a/2h)^2 \\ &\times \int_0^T dt' \;\; m(t'+\tau)\, m^*(t') \\ &\times \int_0^{2\pi} d\theta_T \; \left\{ \frac{\rho_T}{r_T^2\, r_R^2} \,|\, p\,(\hat{e}_{TP})\,|^2 \,|\, p\,(\hat{e}_{RP})\,|^2 \right. \\ &\times \quad [1 + (|\hat{e}_{PT} + \hat{e}_{PR}|^2 - \nu_z^2)/\nu_z^4 \\ &\qquad - 2(|\hat{e}_{PT}+\hat{e}_{PR}|^2 - \nu_z^2)/\nu_z^2]\; \exp(-i\,\omega_0\,\alpha_P\,\tau) \\ &\times \quad \left. \exp[-(a/2\,h\nu_z)^2\, |\hat{e}_{PT}+\hat{e}_{PR}|^2]\,(-d\rho_T/dt') \right\}_{(t,\theta_T,t')} \end{aligned} \tag{75c}$$

where the integrand is evaluated at the ellipse-point $(t,\, \theta_T,\, t')$. Also, since in the exterior region the shading correlation function is approximately independent of θ_T and t', it is denoted simply as a function of t.

Two useful expressions for computing the integrand are the ellipse relation and the range relation, respectively.

$$r_T + r_R = c_0(t+t_D-t') \tag{76a}$$

$$\vec{r}_T + \vec{r}_R = \vec{R} + (D_R-D_T)\,\hat{k} \tag{76b}$$

where the quantity $\vec{R}$ is the horizontal range vector from the transmitter to the receiver and the quantities D_R and D_T are the distances to the bottom from the transmitter and receiver.

With appropriate transpositions each expression is squared in equation 76 and combined to obtain the range expressions

$$r_T = \frac{c_0^2(t+t_D-t')^2 - [R^2+(D_R-D_T)^2]}{2\left\{c_0(t+t_D-t')^2 - \hat{e}_{TP} \cdot [\vec{R}\,(D_R-D_T)\hat{k}]\right\}} \tag{77a}$$

$$r_R = \frac{c_0^2(t+t_D-t')^2 - [R^2+(D_R-D_T)^2]}{2\left\{c_0(t+t_D-t') + \hat{e}_{RP} \cdot [\vec{R}\,(D_R-D_T)\hat{k}]\right\}} \tag{77b}$$

The first relation gives the transmitter range coordinate to the ellipse point. Expressions are obtained for the receiver range and polar coordinates to the ellipse point by first substituting equation 77 into the ellipse relation in equation 76a to obtain the polar coordinate relation

$$\hat{e}_{RP} \cdot [\vec{R}+(D_R-D_T)\hat{k}] = \left\{\hat{e}_{TP} \cdot [\vec{R}+(D_R-D_T)\hat{k}] - \frac{2\,c_0(t+t_D-t')\,[R^2+(D_R-D_T)^2]}{\left\{c_0^2(t+t_D-t')^2 + [R^2+(D_R-D_T)^2]\right\}}\right\}$$

$$\times \left\{1 - \frac{2\,c_0(t+t_D-t')\,\hat{e}_{TP} \cdot [\vec{R}+(D_R-D_T)\hat{k}]}{\left\{c_0^2(t+t_D-t')^2 + [R^2+(D_R-D_T)^2]\right\}}\right\} \tag{78a}$$

and then substituting into equation 77b to obtain the range relation

$$r_R = \frac{\left\{c_0^2(t+t_D-t')^2 - 2\,c_0(t+t_D-t')\,\hat{e}_{TP} \cdot [\vec{R}+(D_R-D_T)\hat{k} + [R^2+(D_R-D_T)^2]\right\}}{2\left\{c_0(t+t_D-t') - \hat{e}_{TP} \cdot [\vec{R}+(D_R-D_T)\hat{k}]\right\}}\,. \tag{78b}$$

With these coordinate relations the integrand in equation 75c can be evaluated.

The derivative of the transmitter cylindrical polar coordinate is given by the derivative of the transmitter range coordinate

$$\rho_T \frac{d\rho_T}{dt'} = \frac{1}{2}\frac{d\rho_T^2}{dt'} = \frac{1}{2}\frac{d}{dt'}(r_T^2+D_T^2) = r_T \frac{dr_T}{dt'} \tag{79a}$$

and the range derivative expression follows by direct computation

$$\begin{aligned}-\left(\frac{1}{r_T r_R^2}\frac{dr_T}{dt'}\right) &= 4c_0[c_0(t+t_D-t') - \hat{e}_{TP}\cdot[\vec{R}+(D_R-D_T)\hat{k}]] \\ &\times\left\{c_0^2(t+t_D-t')^2 - [R^2+(D_R-D_T)^2]\right\}^{-1} \\ &\times\left\{c_0^2(t+t_D-t')^2 - 2c_0(t+t_D-t')\,\hat{e}_{TP}\cdot[\vec{R}+(D_R-D_T)\hat{k}]\right. \\ &\left.+[R^2+(D_R-D_T)^2]\right\}^{-1}\end{aligned} \tag{79b}$$

where the range relations in equation 77f and equation 78b were used.

The scattering correlation function contains the quantities

$$\begin{aligned}|\hat{e}_{PT}+\hat{e}_{PR}|^2 &= 2[1+\hat{e}_{PT}\cdot\hat{e}_{PR}] \\ &= 2+[r_T^2+r_R^2-D^2]/r_T r_R\end{aligned} \tag{80a}$$

$$\nu_z = D_T/r_T + D_R/r_R \tag{80b}$$

which can be evaluated using the preceding range relations. However, since in the far field these functions are nearly constant in θ_T, it will be convenient to use the far field limits $|\hat{e}_{PT}+\hat{e}_{PR}|^2 \to 4$, $\nu_z \to 0$, $r_T \sim r_R \to \infty$.

Substituting into the correlation function and considering, for simplicity, the case of $R >> D_R - D_T$, then

$$
\begin{aligned}
C(t+\tau,t) \; = \; & \frac{C_0 \; C_R(0) \; C_\Pi(t)}{4\pi} \left(\frac{a}{h}\right)^2 \; \exp(a/2h)^2 \\
& \times \int_0^T dt' \;\; m(t'+\tau) \;\; m^*(t') \\
& \times \int_0^{2\pi} d\theta_T \Big\{ |p(\hat{e}_{TP})|^2 \;\; |p(\hat{e}_{RP})|^2 \exp(-i \;\; \omega_0 \; \alpha_P \; \tau) \\
& \times \quad [1 + (2/\nu_z)^4 - 2(2/\nu_z)^2] \; [c_0(t+t_D-t') - \hat{e}_{TP} \cdot \vec{R}] \\
& \times \quad [c_0^2(t+t_D-t') - R^2]^{-1} \; [c_0^2(t+t_D-t') \\
& \qquad -2 \; c_0(t+t_D-t') \; \hat{e}_{TP} \cdot \vec{R} + R^2]^{-1} \\
& \times \;\; \exp[-(a/h \; \nu_z)^2] \Big\}_{(t,\theta_T,t')} \quad . \qquad (81)
\end{aligned}
$$

Further the case of times is considered much greater than the pulse length $t+t_D >> t'$ and the integrals in the correlation function uncouple

$$
\begin{aligned}
C(t+\tau,t) = {} & \frac{c_0\, C_R(0)\, C_\Pi(t)}{4\pi} \left(\frac{a}{h}\right)^2 \frac{\exp (a/2h)^2}{[c_0^2(t+t_D)-R^2]} \\
& \times \left\{ \int_0^T dt' \;\; m(t'+\tau)\, m^*(t') \right\} \\
& \times \left\{ \int_0^{2\pi} d\theta_T \Big(|p(\hat{e}_{TP})|^2 \; |p(\hat{e}_{RP})|^2 \exp(-i\,\omega_0\,\alpha_P\,\tau) \right. \\
& \times \; [1 + (2/\nu_z)^4 - 2(2/\nu_z)^2]\, [c_0(t+t_D) - \hat{e}_{TP}\cdot\vec{R}] \\
& \times \; [c_0^2(t+t_D)^2 - 2\, c_0(t+t_D)\, \hat{e}_{TP}\cdot\vec{R} + R^2]^{-1} \\
& \left. \times \; \exp[-(a/h\,\nu_z)^2] \Big)_{(t,\theta_T)} \right\} . \qquad (82)
\end{aligned}
$$

These approximations reduce the correlation function to a simple form from which, in the following section, a Fourier transformation will give the GIPS of the reverberant scattering.

4. TIME EVOLVING SPECTRUM

GENERALIZED INSTANTANEOUS POWER SPECTRUM (GIPS)

The GIPS is the time varying power spectrum of a nonstationary process. It is related to the nonstationary correlation function of the process by the Fourier transform

$$
P(t,\omega) = \mathrm{Re} \int_{-\infty}^{\infty} C(t+\tau,t) \exp(i\,\omega\,\tau)\, d\tau \qquad (83)
$$

and this result, which is developed elsewhere,[12] is the extension of the Wiener-Kintchine theorem for stationary processes. It resolves the time dependent power of the process into time varying spectral components and can be estimated by the sliding FFT calculation.[12]

Early Arriving Signal

The early arriving signal is a mixture of the direct path signal and the specular scattered signal. The GIPS of this signal is given by the Fourier transform of equation 58 where the correlation function of the specular scattering is given in equation 73c. The result of this calculation is approximated by

$$P(t,\omega) = \frac{Re}{r^2} |p(\hat{e}_{TR})|^2 \; |p(\hat{e}_{RT})|^2 \; m(t) \exp\left[i\,(\omega-\omega_0\,\alpha_D)t\right] \; M^*(\omega-\omega_0\,\alpha_D)$$

$$\times \left\{ 1 + C_\Pi(0,\vec{r}_{TSP})\, C_R(0)\, (a/4h)^2 \exp[i\;\omega_0 t(\alpha_D-\alpha_{SP}) - i(\omega-\omega_0\,\alpha_{SP})\,\tau_{SP}] \right\} \qquad (84)$$

where beam pattern and path length differences have been ignored between the direct path signal and bottom scattered signal and the mean specular scattering in equation 50a is assumed to be negligible due to the exponential function.

This expression shows that the GIPS is the sum of two functions. The first is the GIPS of the direct path signal; the second is the GIPS of the direct path signal which has the Lloyd mirror effect modulation impressed upon it. The amplitude of this modulation depends on the bottom characteristics, but for typical conditions can be expected to be of the order of unity, i.e., the Lloyd mirror modulation produces surface variations that are approximately 50 percent of the GIPS values.

Late Arriving Signal

The GIPS for the late arriving signal is given by the Fourier transform of equation 82. Since this expression is a product of two functions of the transform variable, there is obtained from the the convolution theorem

$$P(t,\omega) = \frac{c_0\, C_R(0)\, C_\Pi(t)}{4\pi} \left(\frac{a}{h}\right)^2 \frac{\exp\,(a/2h)^2}{[c_0^2(t+t_D)-R^2]} \; [P(\omega) * Q(t,\omega)\,] \qquad (85a)$$

$$P(\omega) = F_\tau \left\{ \int_0^T dt'\; m(t'+\tau)\; m^*(t') \right\} \qquad (85b)$$

$$Q(t,\omega) = F_\tau \left\{ \int_0^{2\pi} d\theta_T \left(|p(\hat{e}_{TP})|^2 \; |p(\hat{e}_{RP})|^2 \exp(-i\,\omega_0\,\alpha_P\,\tau) \right.\right.$$

$$\times \; [1 + (2/\nu_z)^4 - 2(2/\nu_z)^2] \; [c_0(t+t_D) - \hat{e}_{TP} \cdot \vec{R}]$$

$$\times \; [c_0^2(t+t_D) - 2\,c_0(t-t_D)\,\hat{e}_{TP} \cdot \vec{R} + R^2]^{-1}$$

$$\left.\left. \times \; \exp[-(a/h\,\nu_z)^2] \right)_{(t,\theta_T)} \right\} \tag{85c}$$

where the asterisk denotes convolution and the symbol $F_\tau \;\{\cdot\}$ denotes Fourier transformation wrt the "τ" variable. The first Fourier transform, $P(\omega)$, is the signal modulation power spectrum, while the second is readily found to be

$$Q(t,\omega) = 2\pi \int_0^{2\pi} d\theta_T \left(|p(\hat{e}_{TP})|^2 \; |p(\hat{e}_{RP})|^2 \; \delta(\omega-\omega_0\,\alpha_P) \right.$$

$$\times \; [1 + (2/\nu_z)^4 - 2(2/\nu_z)^2] \; [c_0(t+t_D) - \hat{e}_{TP} \cdot \vec{R}]$$

$$\times \; [c_0^2(t+t_D)^2 - 2\,c_0(t+t_D)\,\hat{e}_{TP} \cdot \vec{R} + R^2]^{-1}$$

$$\left. \times \exp[-(a/h\,\nu_z)^2] \right)_{(t,\theta_T)} . \tag{86}$$

Inspection of this integral shows that the delta function contributes only for those angles satisfying the condition

$$\omega - \omega_0\,\alpha_P = 0 \;, \tag{87a}$$

i.e., the movements of the transmitter and receiver platforms generate a mapping between angle and frequency. Denoting the direction of transmitter and receiver travel to be α_T and α_R, respectively, then

$$\frac{v_T}{c_0}\cos(\theta_T - \alpha_T) + \frac{v_R}{c_0}\cos(\theta_R - \alpha_R) = \frac{\omega-\omega_0}{\omega_0} \; . \tag{87b}$$

This expression is valid for the far field in which the vertical deflections can be ignored and the transmitter and receiver polar angles are related through equation 78a.

This relation shows that the energy, transmitted into the angle θ_T and received from the angle θ_R, is mapped into a limited band of frequencies. Further, it shows that the energy, transmitted and received via directions that are symmetrical about the platform movements, are mapped into the same frequencies; i.e., defining the symmetrical directions to be θ_T' and θ_R' then

$$\alpha_T - \theta_T' = \theta_T - \alpha_T \tag{88a}$$

$$\alpha_R - \theta_R' = \theta_R - \alpha_R \quad . \tag{88b}$$

Consequently equation 87b will have two sets of angular roots (θ_T, θ_R) for each frequency. The times of arrival corresponding to each set of roots must satisfy equation 78a and, generally, will not be the same. However, simple geometric arguments can be constructed to show that equation 87b will always have a double root system. In the long time limit, $t \to \infty$, equation 78a yields the limit $\theta_R \to \theta_T$. Equation 87b shows that in this limit the angles corresponding to the maximum and minimum frequencies are separated by π radians. The axis that separates these angles is designated as the ambiguity axis. The integral in equation 86 separates into two terms corresponding to the contributions from each side of the ambiguity axis at each frequency

$$Q(t,\omega) = Q^{(1)}(t,\omega) + Q^{(2)}(t,\omega) \tag{89a}$$

$$\begin{aligned} Q^{(\ell)}(t,\omega) = {} & 2\pi \, |p(\hat{e}_{TP})|^2 \, |p(\hat{e}_{RP})|^2 \\ & \times [1 + (2/\nu_z)^4 - 2(2/\nu_z)^2] \, [c_0(t+t_D) - \hat{e}_{TP} \cdot \vec{R}] \\ & \times [c_0^2(t+t_D)^2 - 2\, c_0(t+t_D)\, \hat{e}_{TP} \cdot \vec{R} + \vec{R}^2]^{-1} \\ & \times \exp[-(a/h\,\nu_z)^2] \quad . \end{aligned} \tag{89b}$$

Collecting terms the GIPS in equation 85a becomes

$$\begin{aligned} P(t,\omega) = {} & \frac{c_0\, C_R(0)\, C_\Pi(t)}{4\pi} \left(\frac{a}{h}\right)^2 \frac{\exp(a/2h)^2}{[c_0^2(t+t_D)^2 - R^2]} \\ & \times P(\omega) * [Q^{(1)}(t,\omega) + Q^{(2)}(t,\omega)] \end{aligned} \tag{90}$$

which shows that the power at each time and frequency is contributed from both sides of the ambiguity axis. Each contribution is amplitude weighted by the beam pattern and transmission path associated with the respective side of the ambiguity axis and the power is smoothed in frequency via a convolution with the transmit power spectrum. If the bandwidth of the transmitted signal is small compared to the band of Doppler shifted frequencies then it is approximated

$$P(\omega) = 2\pi\,\delta(\omega)$$

and the GIPS obtained

$$P(t,\omega) = c_0\, C_R(0)\, C_\Pi(t) \left(\frac{a}{h}\right)^2 \frac{\exp{(a/2h)^2}}{[c_0^2(t+t_D)-R^2]} \times [Q^{(1)}(t,\omega) + Q^{(2)}(t,\omega)]/2 \quad . \tag{92}$$

This is our principal representation of the far field GIPS which, to be complete, needs an explicit angle-frequency mapping relation.

An exact angle-frequency mapping relation for the far field can be obtained from equation 78a and equation 87b but for these purposes it is instructive and useful to obtain a longtime, $t \to \infty$, zeroth order approximation. Substituting this approximtion of equation 78a,

$$\cos(\vartheta_R-\Gamma) = \cos(\theta_T-\Gamma) \quad , \tag{93}$$

which is exact in the long time limit, into 87b, the angle-frequency mapping obtained is

$$\cos(\theta_T-\Gamma) = \frac{B}{A^2}\left(\frac{\omega-\omega_0}{\omega_0}\right) \pm \frac{C}{A^2}\left[A^2 - \left(\frac{\omega-\omega_0}{\omega_0}\right)^2\right]^{1/2} \tag{94a}$$

$$A = [v_T^2 + 2v_T v_R \cos(\alpha_T-\alpha_R) + v_R^2]^{1/2}/c_0 \tag{94b}$$

$$B = [v_T \cos(\Gamma-\alpha_T) + v_R \cos(\Gamma-\alpha_R)]/c_0 \tag{94c}$$

$$C = [v_T \sin(\Gamma-\alpha_T) + v_R \sin(\Gamma-\alpha_R)]/c_0 \tag{94d}$$

where Γ is the angle to the range vector $\vec{R}$ and where the constants A, B, C are strictly functions of the track parameters. The constant A gives the maximum range of fractional Doppler shift in frequency, B gives the fractional Doppler shift in frequency due to the relative range rate and C gives the fractional Doppler shift due to the relative transversal rate. In principle these constants are measurable, since A and B can be estimated from spectral data and they are related to C through the relation

$$A^2 = B^2 + C^2 \quad . \tag{94e}$$

Further, the angle-frequency mapping shows explicitly that two angles are mapped into each frequency in the Doppler band $\omega_0(1 \pm A)$; i.e., the spectral density at any frequency in general cannot be unambiguously resolved into components associated with the directions of transmission. The direction of the ambiguity axis corresponds to the upper and lower edges of the Doppler band and is given by

$$\theta_T^A = \Gamma + \cos^{-1}(B/A) \quad \cdot \tag{94f}$$

These mapping relations can be derived more accurately by using higher order approximations of equation 78a.

SPECTRAL VARIANCE

The variance of the spectral ensemble provides an estimate of the usefulness of any one spectral estimate and a bound on this function is given by[10]

$$\begin{aligned} \Delta^2 P(t,\omega) \leqslant \iint_{-\infty}^{\infty} d\tau d\tau' \Big\{ & E\ \phi(t+\tau)\ \phi^*(t) - C(t+\tau,t) \Big] \\ & \times [\ \phi^*(t+\tau')\ \phi(t) - C^*(t+\tau',t)] \Big\} \\ & \times \exp[i\ \omega(\tau-\tau')] \quad . \end{aligned} \tag{95}$$

For the case of ocean bottom reverberation the signal can be approximated as a Gaussian random variable and using the Gaussian moment expression

$$E[x_1x_2x_3x_4] = E[x_1x_2]\,E[x_3x_4]$$

$$+E[x_1x_3]\,E[x_2x_4]$$

$$+E[x_1x_4]\,E[x_2x_3]$$

$$-2\,E[x_1]\,E[x_2]\,E[x_3]\,E[x_4] \qquad (96)$$

the variance expression can be approximated.

For the real signal case the bound readily is obtained

$$\triangle^2 P(t,\omega) \leqslant \ <\phi^2(t)><|M(\omega-\omega_0|^2> + P^2(t,\omega)$$

$$-2 <\phi(t)>^2 \ |<M(\omega-\omega_0)>|^2$$

$$\leqslant \ 2[<\phi^2(t)> \ <|M(\omega-\omega_0)|^2>$$

$$-<\phi(t)>^2 \ |<M(\omega-\omega_0)>|^2] \qquad (97)$$

where

$$P^2(t,\omega) = E|\mathrm{Re}\,\phi(t)\exp(i\,\omega t)\,M^*(\omega-\omega_0)|^2$$

$$\leqslant <\phi^2(t)> \ <|M(\omega-\omega_0)|^2> \ . \qquad (98)$$

Normalizing wrt the GIPS envelope

$$P_E^2(t,\omega) = <\phi^2(t)> \ <|M(\omega-\omega_0)|^2>$$

the coefficient of variation obtained is

$$\frac{\Delta^2 P(t,\omega)}{P_E^2(t,\omega)} \leqslant 2 \left[1 - \frac{< \phi(t) >^2 \; |< M(\omega-\omega_0) >|^2}{<\phi^2(t)> \;\; < |M(\omega-\omega_0)|^2>} \right] \tag{100a}$$

which has the approximate bound

$$\frac{\Delta^2 P(t,\omega)}{P_E^2(t,\omega)} \leqslant 2 \tag{100b}$$

for both the early received signal and the late received signal. Loosely interpreted, this result shows that the most any one spectral estimate is expected to differ from the GIPS is approximately 40 percent of the GIPS value. This variation is comparable to the amplitude of the Lloyd mirror effect modulation of the GIPS in equation 84, which implies that the Lloyd mirror effect cannot be estimated from one realization of the spectrum for the case of a moderately rough bottom.

CONCLUSIONS

The GIPS expressions obtained here for the early received signal and the later received signal are our results of principal interest. They exhibit relations between transmitter and receiver parameters and the received spectral components that can be exploited. In particular, the expressions for the later received signal in equation 89 and equation 92 are a generalization of Halley's original work and show how the transmitter and receiver beam deployment parameters influence the later received signal. These expressions provide insights into ocean information transfer problems.

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APPENDIX: HUYGEN'S PRINCIPLE

It is shown elsewhere* that the acoustic field within a volume V containing harmonic sources and bounded by the surface S (see figure A.1) is given by RWS formulation

$$\phi(\vec{r}) = \phi_0(\vec{r}) + \frac{1}{4\pi} \left\{ \int_V dv\, \phi(\vec{\xi})\, \gamma(\vec{\xi},\vec{r}) + \int_S ds\, \vec{n} \cdot [\phi(\vec{\xi}) \nabla - \nabla \phi(\vec{\xi})] \right\} \chi(\vec{\xi},\vec{r}) \qquad (A.1)$$

where the unit vector $\vec{n}$ is directed into the volume. In this field expression the first term represents the direct path field and the volume integral represents the field due to scattering by the medium inhomogeneities. The surface integral represents the field scattered by the surface.

In the following a theorem by Baker and Copson** is extended and shows that the RWS surface integral produces a null effect within V when the surface field is replaced by the direct path field; i.e., Huygen's principle is demonstrated showing outgoing waves have no effect at a point after propagating past that point.

Consider a rederivation of equation A.1 with the modification that the sources are excluded from V by a surface S′ (see figure A.1). By inspection

$$\phi(\vec{r}) = \frac{1}{4\pi} \left\{ \int_V dv\, \phi(\vec{\xi})\, \gamma(\vec{\xi},\vec{r}) + \left(\int_{S'} ds' + \int_S ds \right) \vec{n} \cdot [\phi(\vec{\xi})\, \nabla - \nabla\, \phi(\vec{\xi})] \right\} \chi(\vec{\xi},\vec{r}) \qquad (A.2)$$

* O. D. Grace, Acoustic Ray Wave Synthesis. NUC TP 408, June 1974 and O. D. Grace, Signal Variation in a Random Medium. NUC TP 474, July 1976

** B. B. Baker and E. T. Copson, The Mathematical Theory of Huygen's Principle, Oxford University Press, 1939.

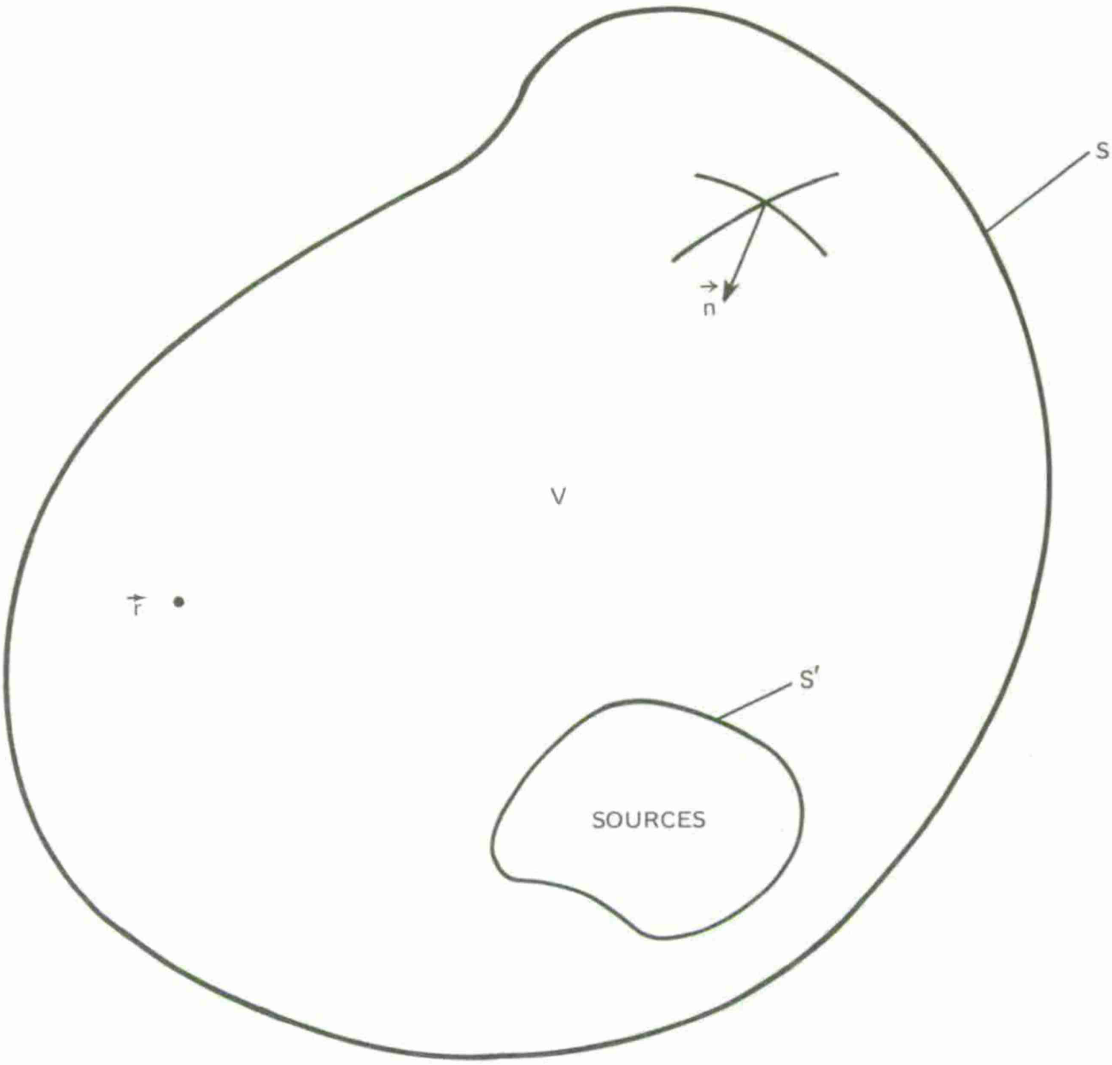

Figure A.1. Acoustic cavity.

where $\vec{n}$ is directed into the volume. Letting the surface S be a spherical shell of radius R tending to infinity then

$$I_S = \lim_{R\to\infty} \int d\omega \; R^2 [\phi(\vec{\xi})\nabla_R \;-\nabla_R \; \phi(\vec{\xi})] \;\; \chi(\vec{\xi},\vec{r}) \tag{A.3}$$

where $d\omega$ is the solid angle subtended by an element of the sphere. For the modified radiation condition[11]

$$\lim_{R\to\infty} R^2 [\phi(\vec{\xi})\nabla_R \;-\nabla_R \; \phi(\vec{\xi})] \; \chi(\vec{\xi},\vec{r}) \;=\; 0 \tag{A.4}$$

this term vanishes and by redefining the unit normal vector to be directed within S′ the "exterior" form of the RWS formulation is obtained, as opposed to the "interior" form given by equation A.1

$$\phi(\vec{r}) \;=\; \frac{1}{4\pi}\left\{\int_V dv \; \phi(\vec{\xi}) \; \gamma(\vec{\xi},\vec{r}) \right.$$

$$\left. - \int_{S'} ds' \;\; \vec{n} \cdot [\phi(\vec{\xi})\nabla - \nabla \; \phi(\vec{\xi})]\right\} \; \chi(\vec{\xi},\vec{r}) \;\; . \tag{A.5}$$

Huygen's principle is demonstrated with the aid of the interior and exterior forms.

Consider the cavity in figure A.2 in which the field point and sources are separated by the imaginary boundary B which divides S into the surfaces S_1 and S_2, i.e., $S = S_1 + S_2$. If S and B are imaginary constructions within the field, then the fields on S and B are given by the direct path field and the field at $\vec{r}$ can be represented by both the interior and exterior forms, respectively

$$\phi(\vec{r}) \;=\; \frac{1}{4\pi}\left\{\int_V dv \; \phi_0(\vec{\xi}) \; \gamma(\vec{\xi},\vec{r}) \right.$$

$$\left. + \int_{S_1+B} ds \; \vec{n} \cdot [\phi_0(\vec{\xi})\nabla - \nabla \; \phi_0(\vec{\xi})] \right\} \chi(\vec{\xi},\vec{r}) \tag{A.6a}$$

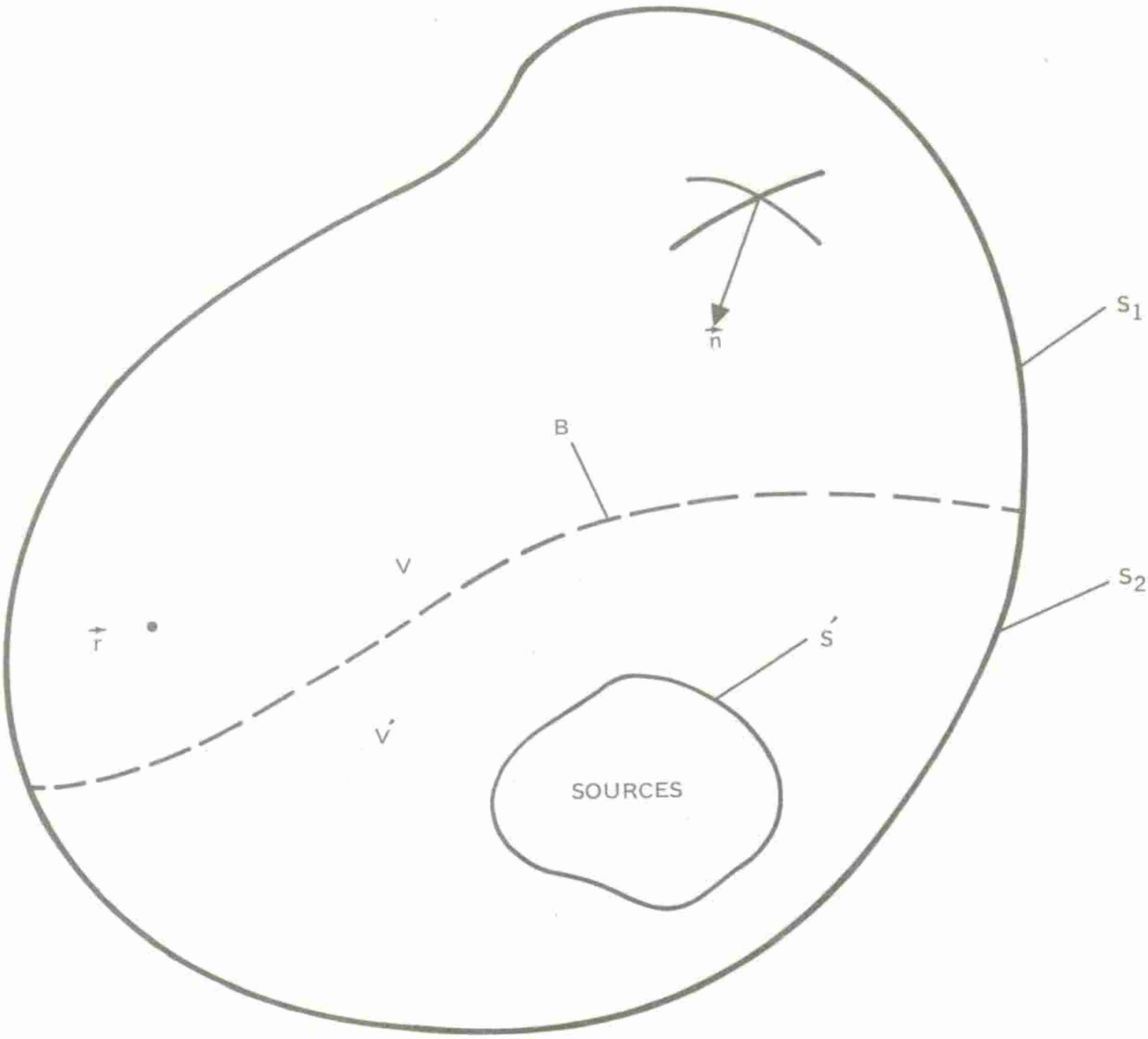

Figure A.2. Constructed surfaces.

$$\phi(\vec{r}) = \frac{1}{4\pi} \left\{ \int_V dv\, \phi_0(\vec{\xi})\, \gamma(\vec{\xi},\vec{r}) \right.$$

$$\left. + \int_{S_2+B} ds\, \vec{n} \cdot [\phi_0(\vec{\xi})\, \nabla - \nabla \phi_0(\vec{\xi})\,] \right\} \chi(\vec{\xi},\vec{r}) \quad . \qquad \text{(A.6b)}$$

Equating expressions the integrals over B are found to cancel, due to opposing unit vectors, and

$$\int_S ds\, \vec{n} \cdot [\phi_0(\vec{\xi})\, \nabla - \nabla\, \phi_0(\vec{\xi})\,] \;\; \chi(\vec{\xi},\vec{r}) = 0 \quad . \qquad \text{(A.7)}$$

This result shows that the surface scattering operator of the RWS formulation, with respect to an arbitrary closed surface S, produces a null effect within S when acting on the incident field, i.e., that Huygen's principle is satisfied.